The cost of being quantum

Or

the rise of the Quadrature Coherent Scale in the Wigner negative valleys of the cubic phase state

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Cubic Gaussian state - Introduction

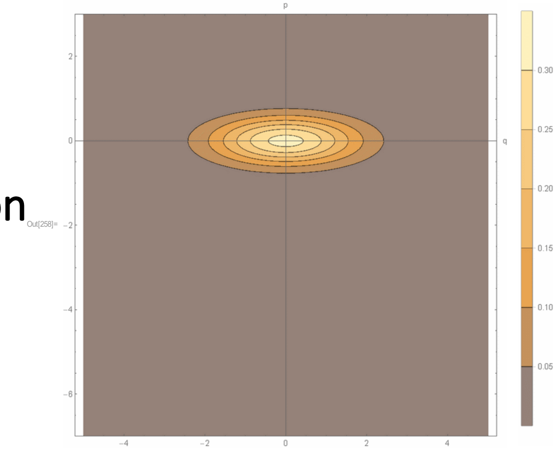
- Ideal Cubic phase state : $|\gamma, p=0\rangle = \hat{\Gamma}_\gamma |p=0\rangle$ → Infinite Wigner negativity, infinite p-squeezing, pure state,

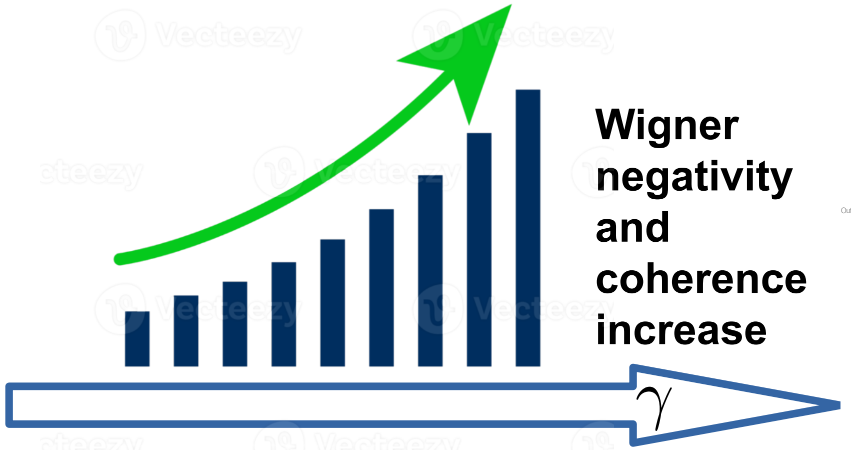
  where $\hat{\Gamma}_\gamma = e^{-i\gamma\hat{q}^3}$

- Cubic squeezed state : $|\gamma, r\rangle = \hat{\Gamma}_\gamma \hat{S}_r |0\rangle$ → Finite Wigner negativity, finite p-squeezing r, pure state
- Cubic Gaussian state : $\hat{\rho}_{G,\gamma} = \hat{\Gamma}_\gamma \hat{\rho}_G \hat{\Gamma}_\gamma^\dagger$ where ρ_G is a Gaussian state : squeezing r and thermal factor q_β

Wigner function

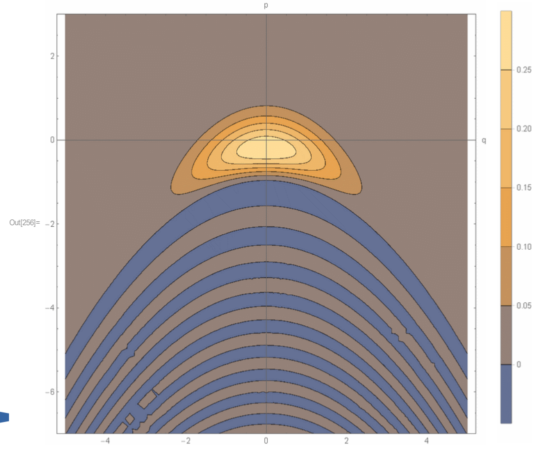
Gaussian





Wigner negativity and coherence increase

Cubic Gaussian state



Cubic Gaussian state – Wigner negativity

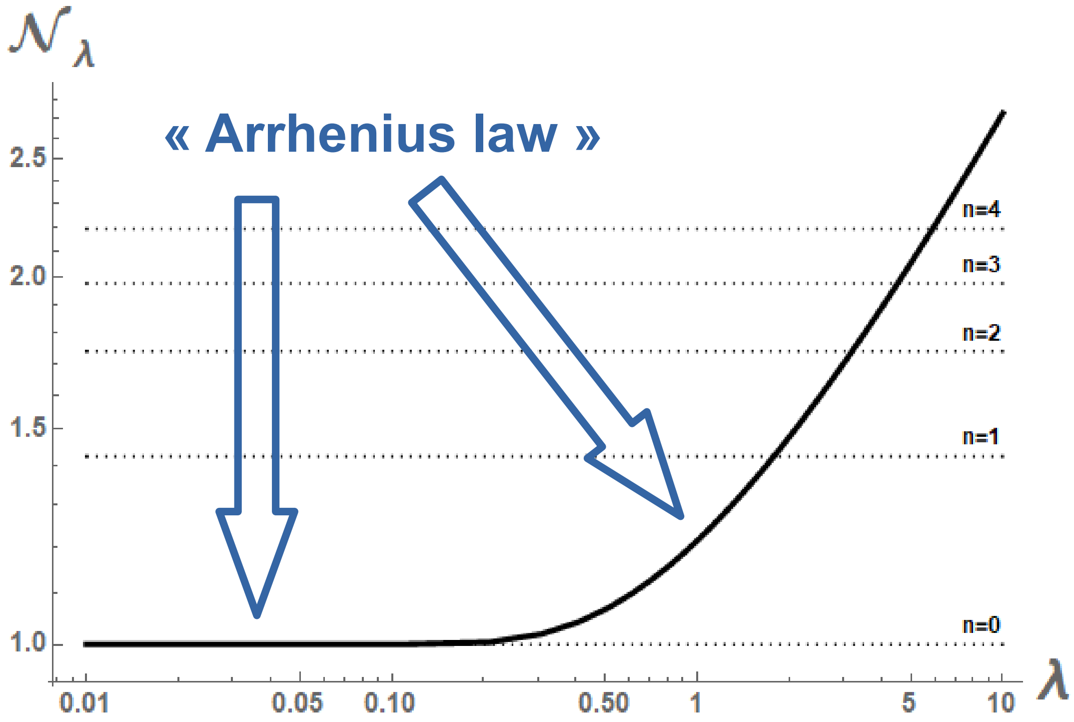
<u>Wigner negativity</u>

$$\mathcal{N}_\rho = \iint |W|(q,p)\,dq\,dp \geq 1$$

Wigner negativity of the cubic Gaussian state

$$\mathcal{N}_{\rho_{G,\gamma}} = \mathcal{N}_\lambda = e^{\frac{-1}{24\lambda^2}} \int e^{\frac{\eta}{2\lambda^{2/3}}} |Ai(\eta)|\,d\eta$$

Only depends on $\lambda = 6\gamma\left(\frac{\sigma_{qq}}{\det V}\right)^{3/2}$



Paying the price for Wigner Negativity [4]

For a state $\hat{\rho}$, the **Quadrature Coherent Scale** (QCS) is defined as :

$$\mathcal{C}^2(\hat{\rho}) = \frac{1}{2\mathcal{P}(\hat{\rho})}\left(Tr[\hat{\rho},\hat{q}][\hat{q},\hat{\rho}] + Tr[\hat{\rho},\hat{p}][\hat{p},\hat{\rho}]\right)$$

Where $\mathcal{P}(\hat{\rho})$ is the purity

Properties:

- States with a **large QCS** are **more sensitive to environmental decoherence** [5]
  - The larger the QCS, the smaller the decoherence time : $\tau_D \simeq \frac{1}{\mathcal{C}^2}$

For the **cubic centered Gaussian state**, its **QCS** is given by an **analytic expression** :

$$\mathcal{C}^2_{\mathrm{G}^0,\gamma} = \left(\frac{1-q_\beta}{1+q_\beta}\right)\cosh(2r) + \frac{9}{2}\gamma^2 e^{4r}.$$

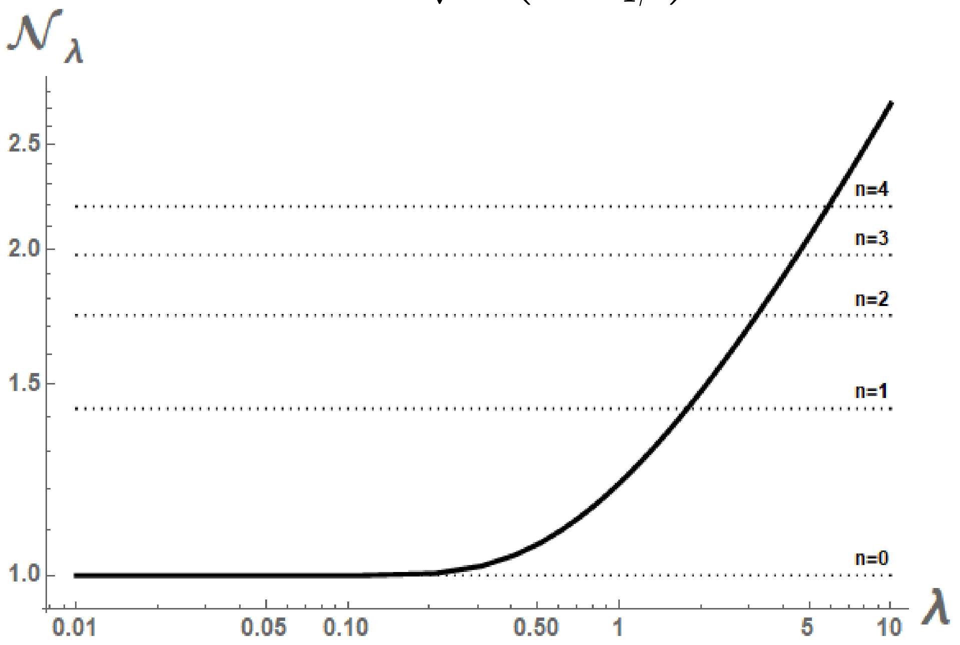
[4] S. De Bièvre, D. B. Horoshko, G. Patera & M. I. Kolobov, Phys. Rev. Lett. **122**, 080402 (2019)
[5] A. Hertz & S. De Bièvre, Phys. Rev. Lett. 124, 090402 (2020)

Minimizing the cost of Wigner Negativity !

For a given Wigner negativity (hence, a given λ), what choice of squeezing and cubicity minimizes the QCS ?

$$\lambda = 6\gamma \left(\frac{\sigma_{qq}}{\det V}\right)^{3/2} = \frac{3}{\sqrt{2}} \left(\frac{1-q_\beta}{1+q_\beta}\right)^{3/2} \gamma \exp(3r)$$

$$\mathcal{C}^2_{\lambda,\min} = \min_{q_\beta, r} \left[\left(\frac{1-q_\beta}{1+q_\beta}\right) \cosh(2r) + \lambda^2 \left(\frac{1+q_\beta}{1-q_\beta}\right)^3 e^{-2r} \right].$$



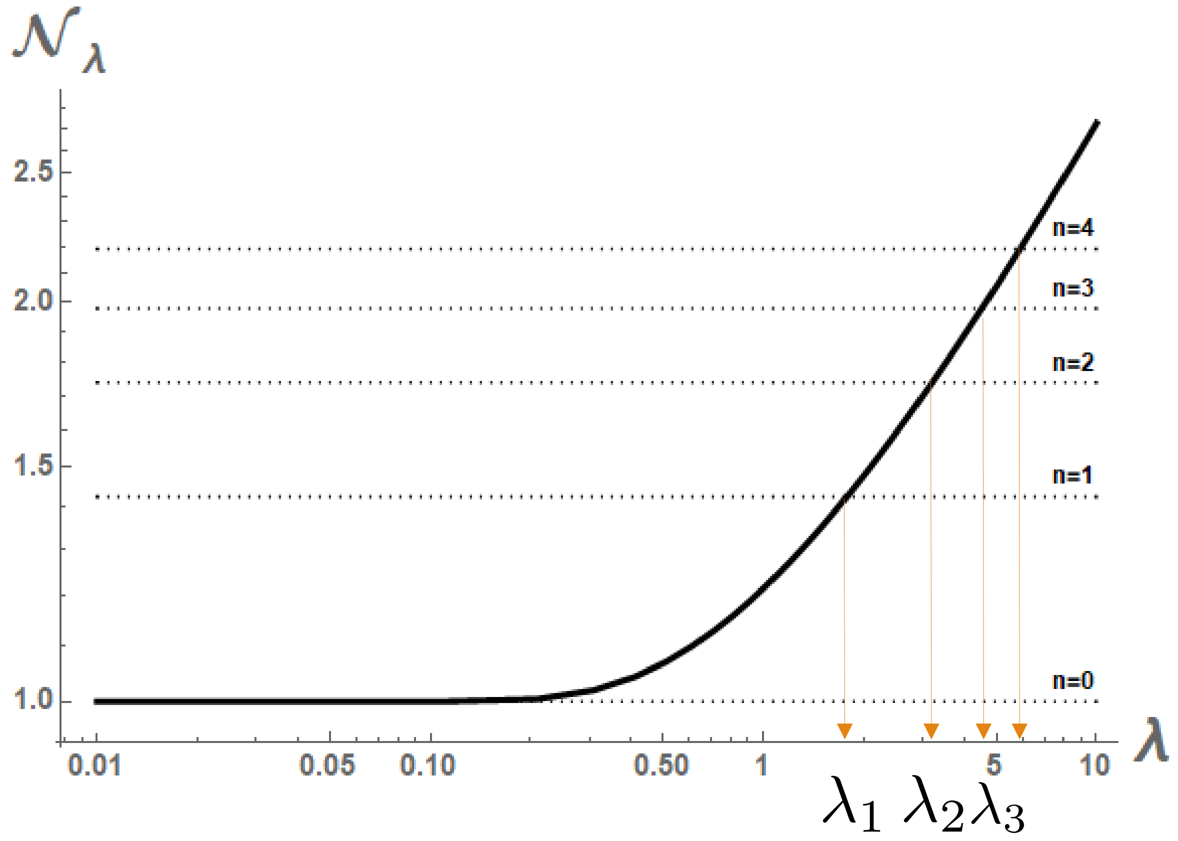
$$r_{\min} = \frac{1}{4} \ln(1+2\lambda^2), \quad q_{\beta,\min} = 0$$

$$\gamma_{\min} = \frac{\sqrt{2}}{3} \frac{\lambda}{(1+2\lambda^2)^{3/4}}$$

$$\mathcal{C}^2_{\lambda,\min} = (1+2\lambda^2)^{1/2}$$

Best offer on the cost of Wigner Negativity for cubic Gaussian state

Is the grass greener on the neighbour's side?



Cubic Gaussian State on the Fock state scale

n	$\mathcal{N}(\lvert n\rangle)$	λ_n	$\mathcal{C}^2(\lvert n\rangle)$	$\mathcal{C}^2_{\lambda_n,\min}(\rho_\gamma)$
0	1.00	0.00	1.00	1.00
1	1.43	1.80	3.00	2.73
2	1.73	3.17	5.00	4.59
3	1.98	4.55	7.00	6.51
4	2.19	5.89	9.00	8.39
5	2.38	7.23	11.0	10.27
6	2.56	8.62	13.0	12.23

$\mathcal{C}^2(\lvert n\rangle) = 2n+1$ $\quad \mathcal{C}^2_{\lambda,\min} = (1+2\lambda^2)^{1/2}$

Conclusion – No free lunch !

If you want some Wigner negativity, you can have it. But, at the end of the day, you pay for it and the more Wigner negativity you want, the more expensive it gets in Quadrature Coherence Scale.