A general framework for (CV) Bell non-locality and contextuality

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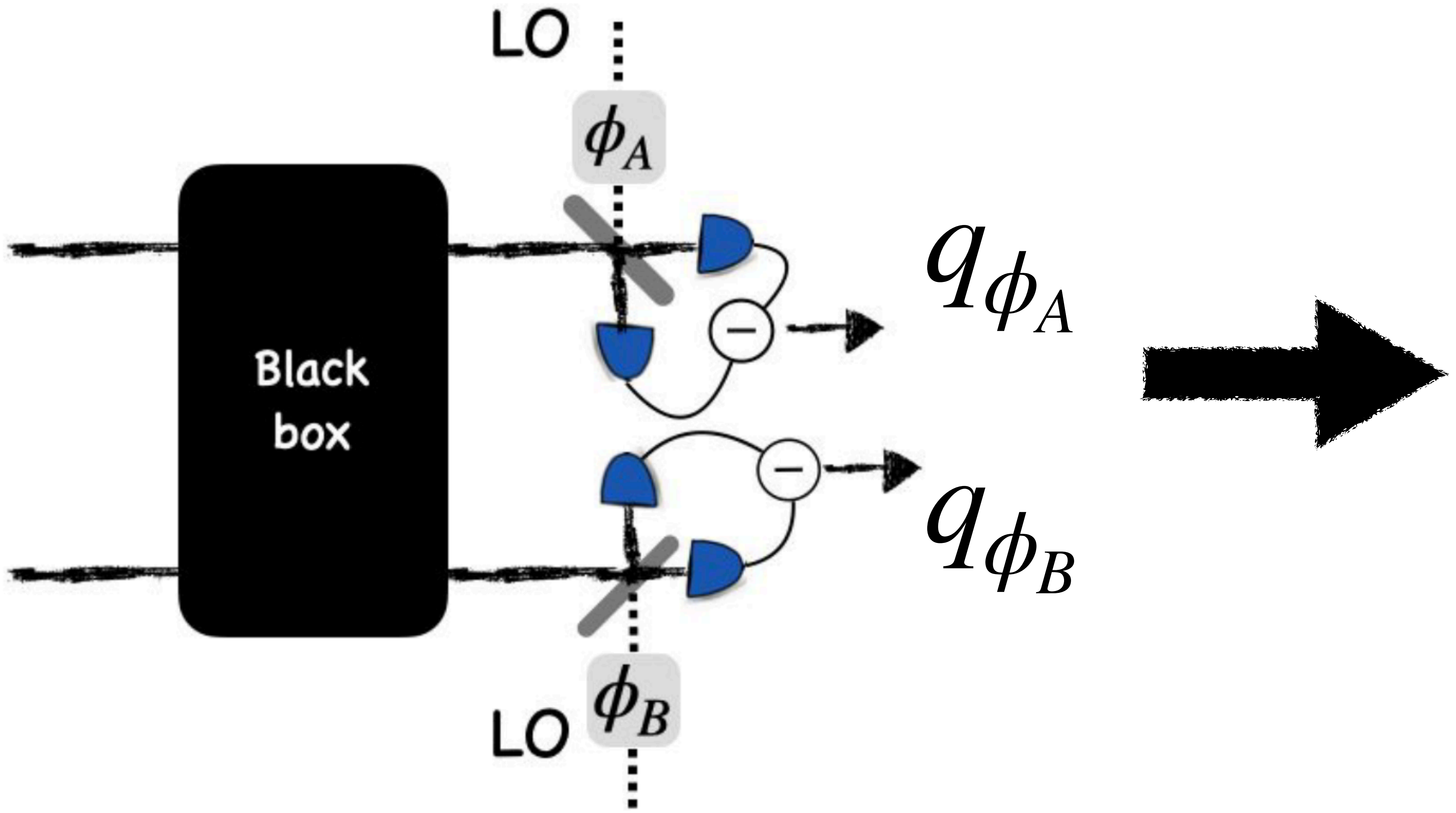
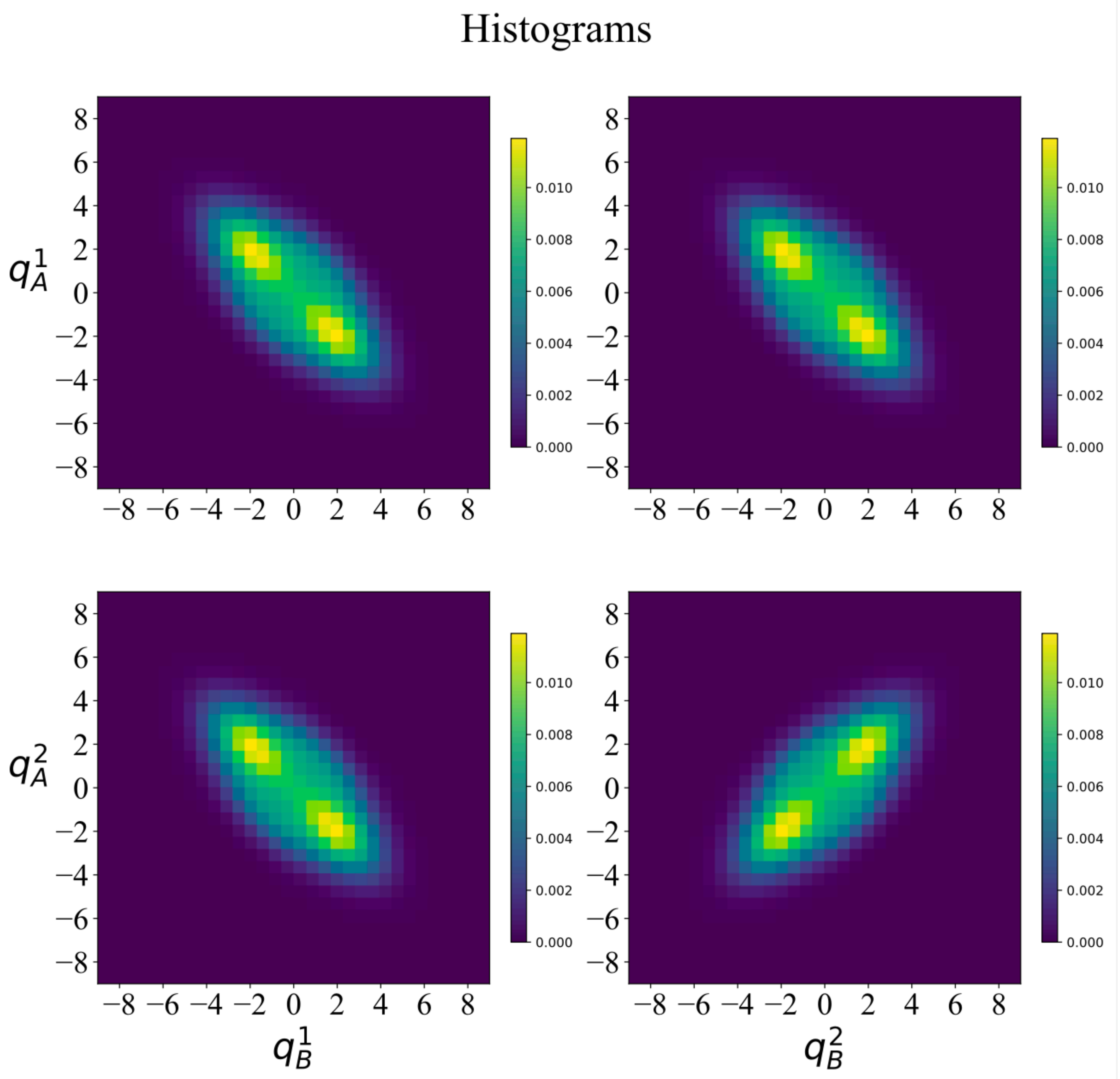
Joint work with: P.E. Emeriau, Ulysse Chabaud, Uta I. Meyer, Enky Oudot, Gael Masse, Frederic Grosshams, Damien Markham, Robert Booth, Federico Centrone and Mattia Walschaers

Objective:

Obtain some genuinely CV and practically relevant Bell inequality





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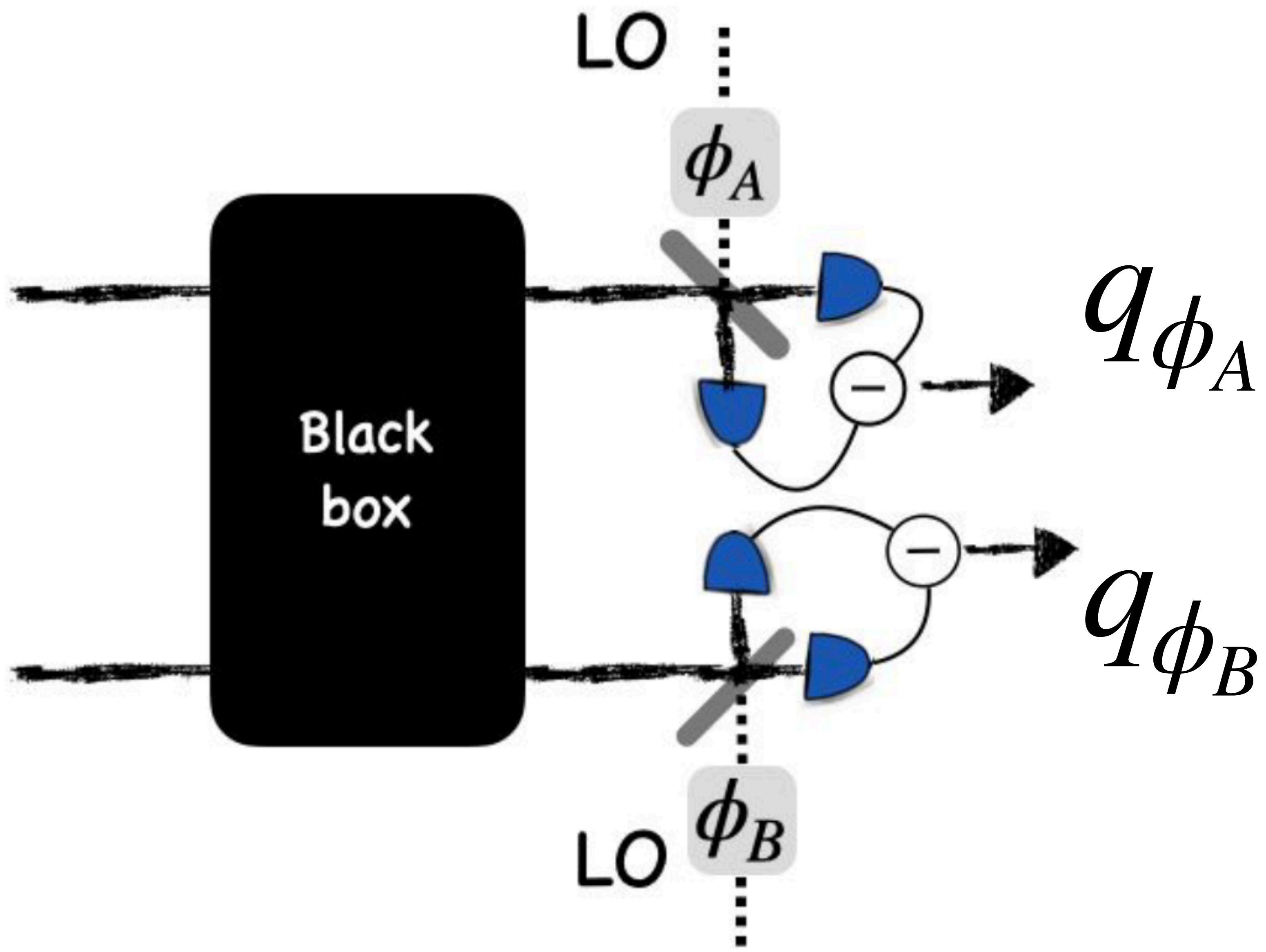

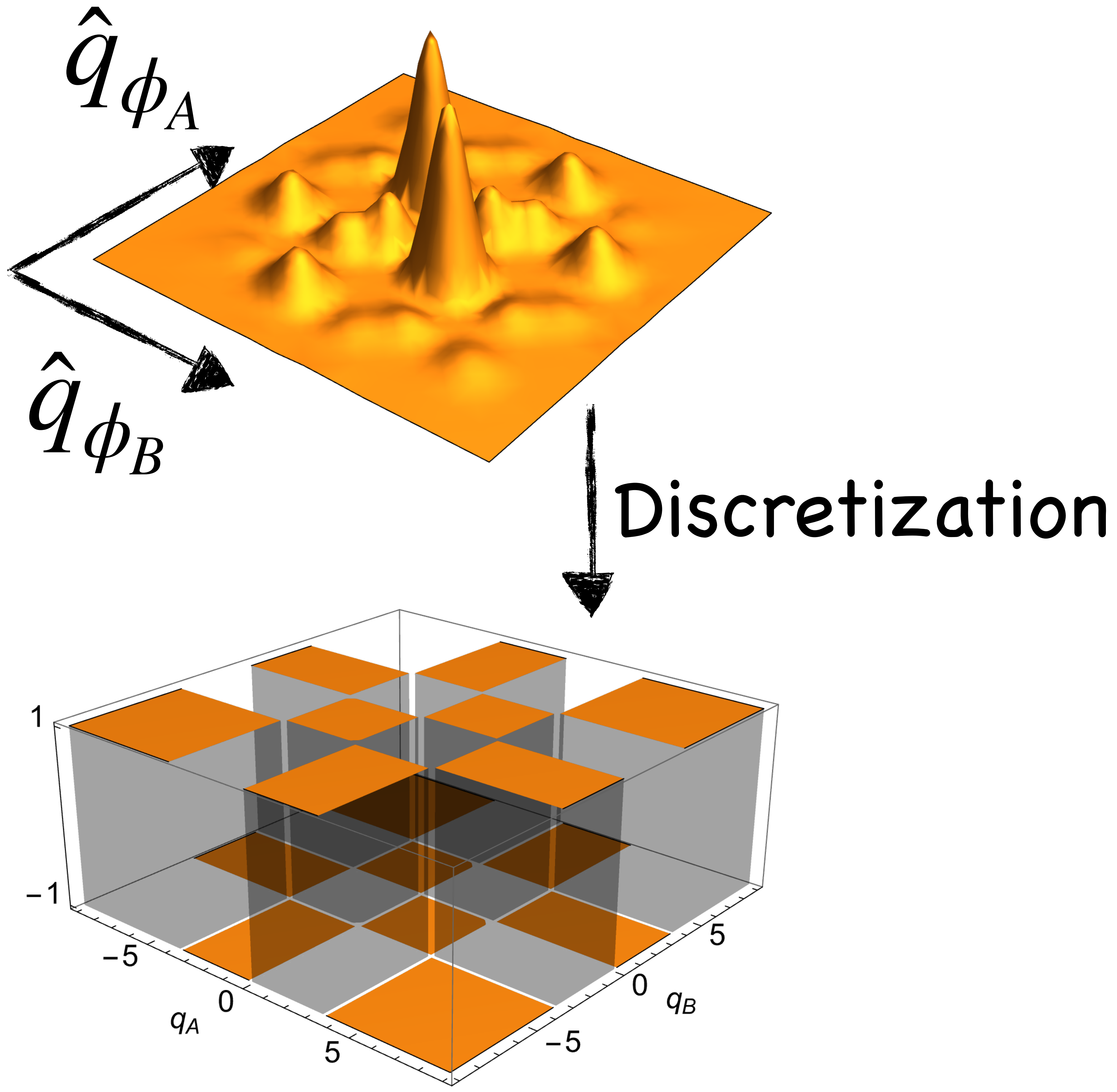

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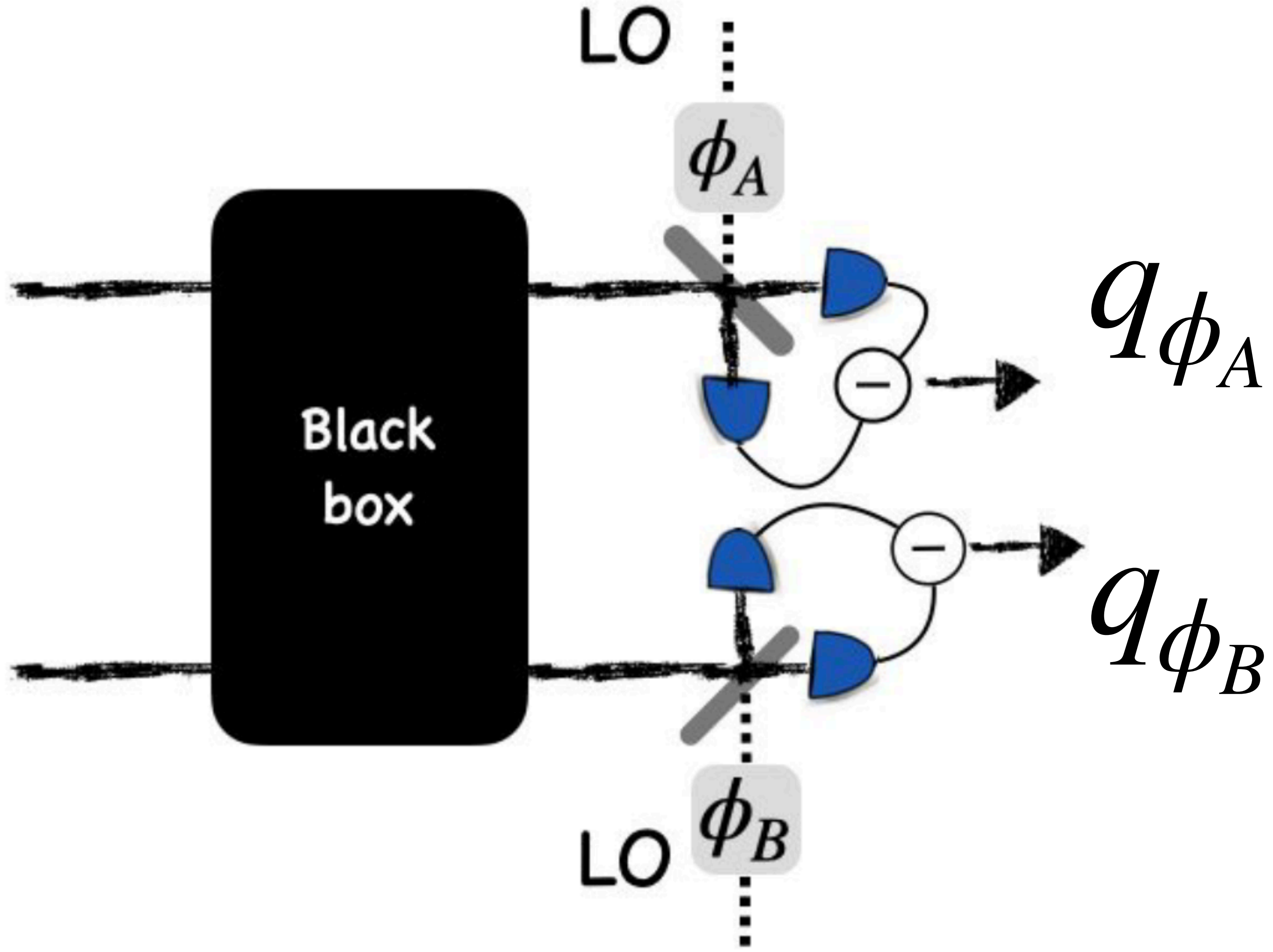
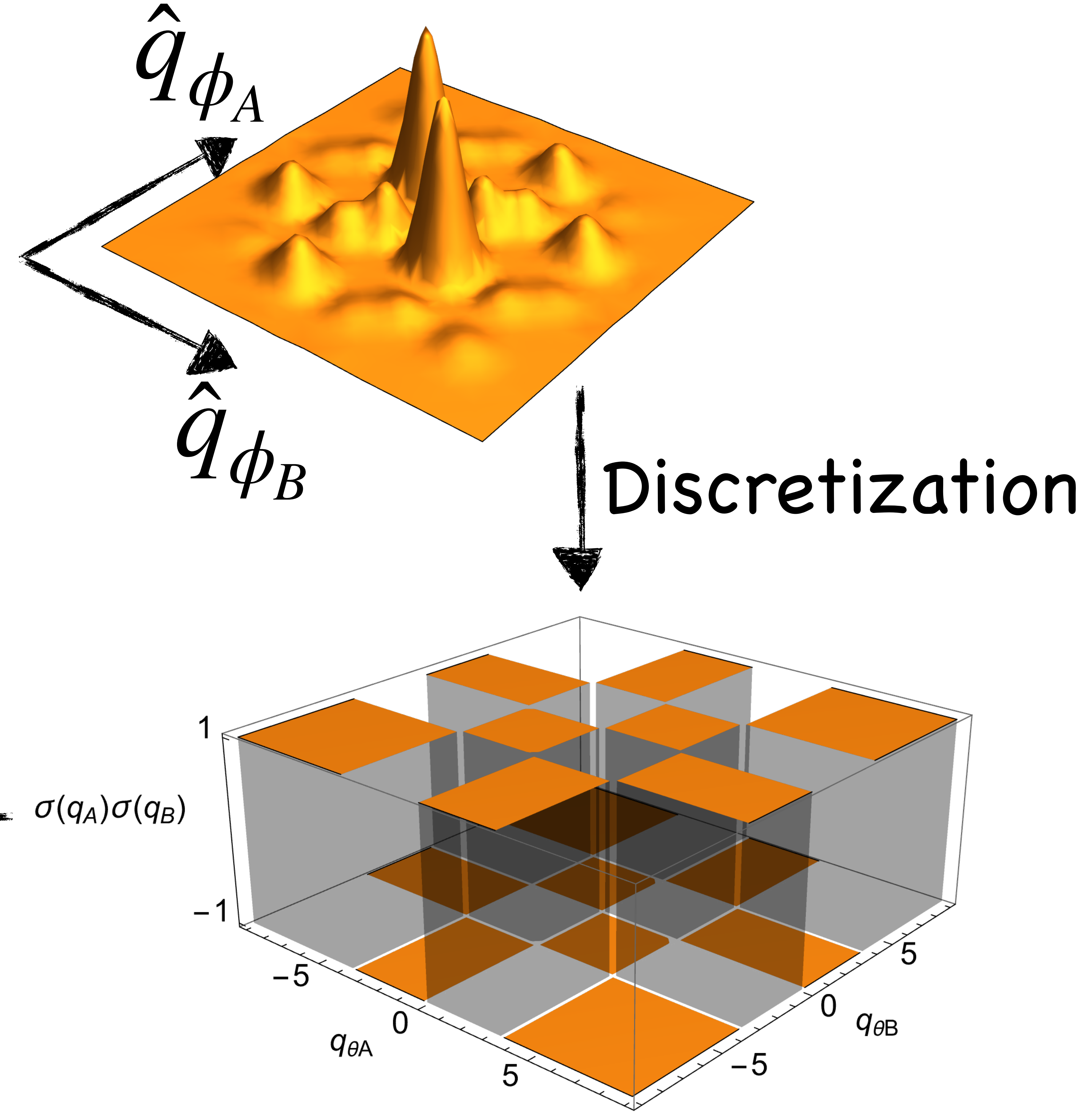




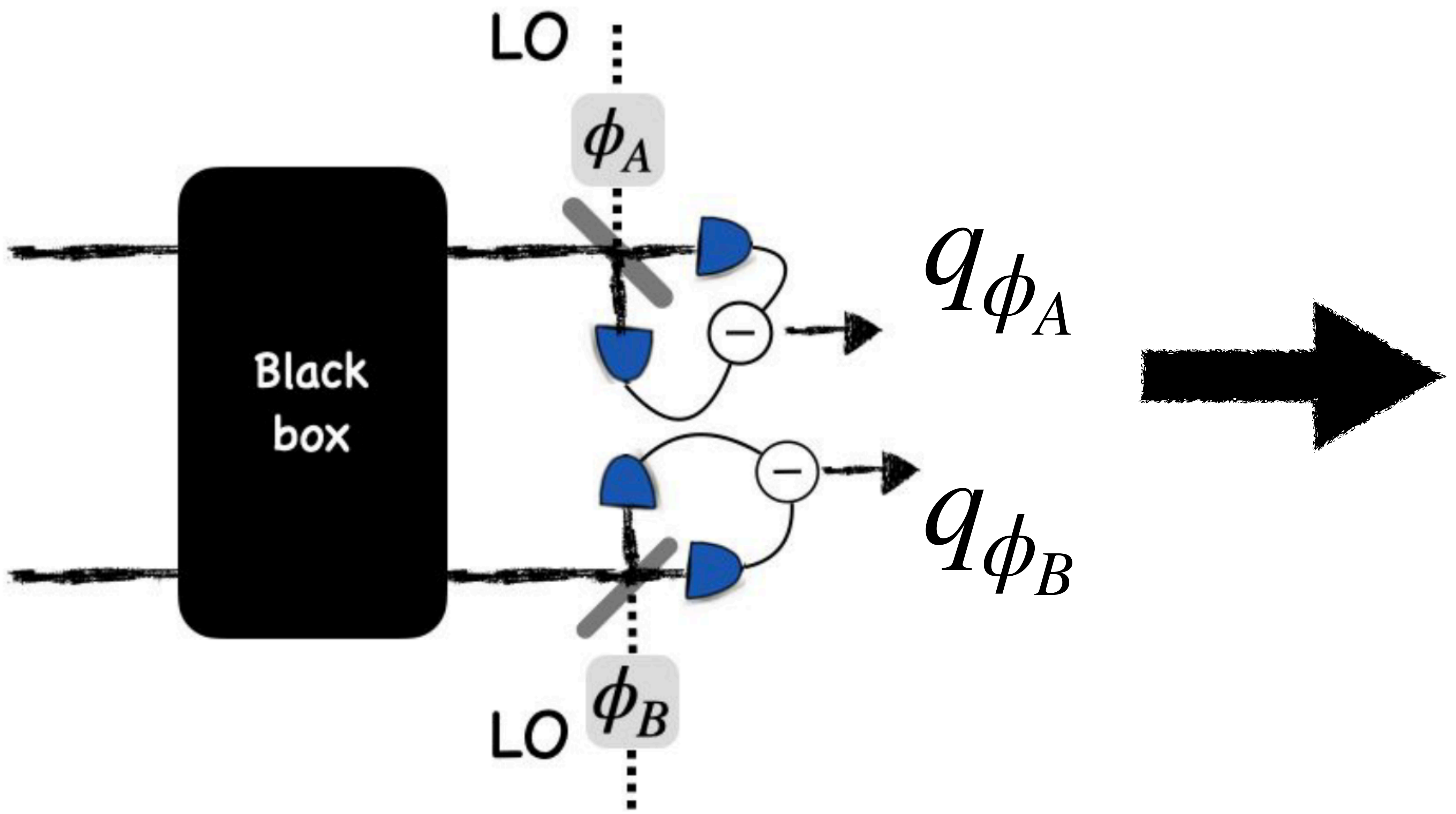
Apply CHSH inequality

$$\langle \sigma_{\phi_A^{(1)}} \sigma_{\phi_B^{(1)}} \rangle + \langle \sigma_{\phi_A^{(1)}} \sigma_{\phi_B^{(2)}} \rangle + \langle \sigma_{\phi_A^{(2)}} \sigma_{\phi_B^{(1)}} \rangle - \langle \sigma_{\phi_A^{(2)}} \sigma_{\phi_B^{(2)}} \rangle \leq 2$$

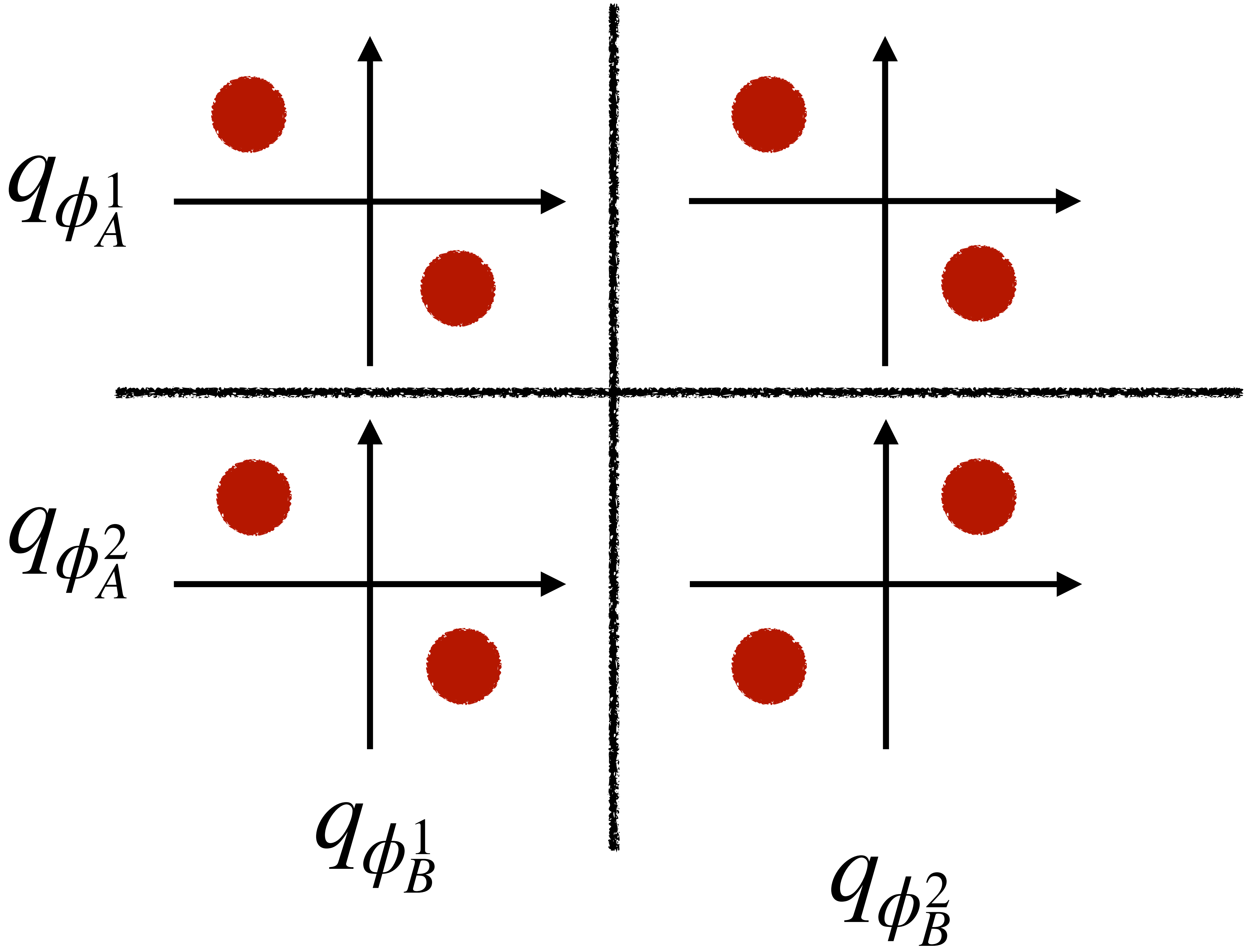
Theorem: For this kind of measurement scenario

Bell non-locality (existence of a LHVM) $\Longleftrightarrow$ Contextuality

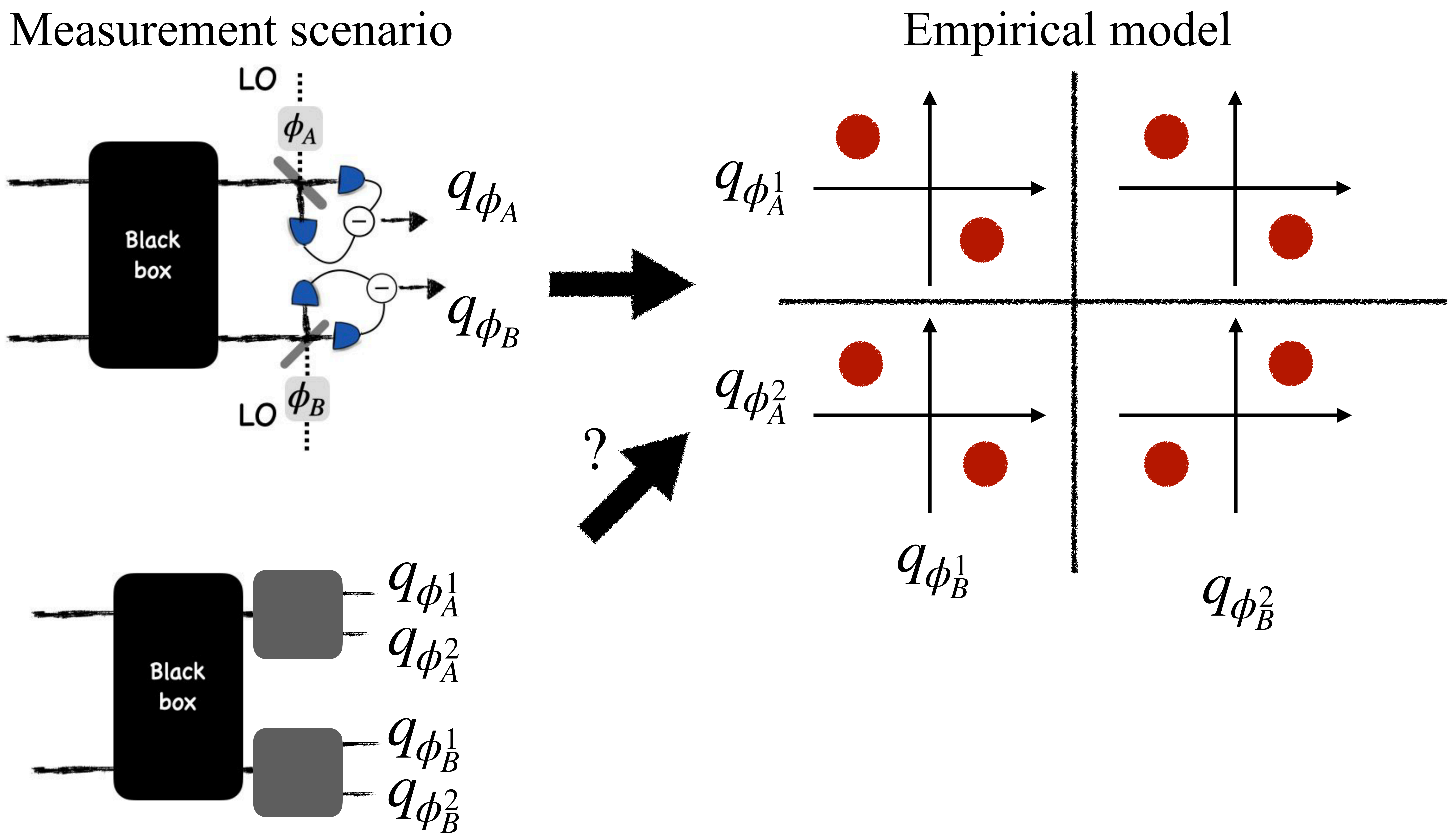
Measurement scenario



Empirical model



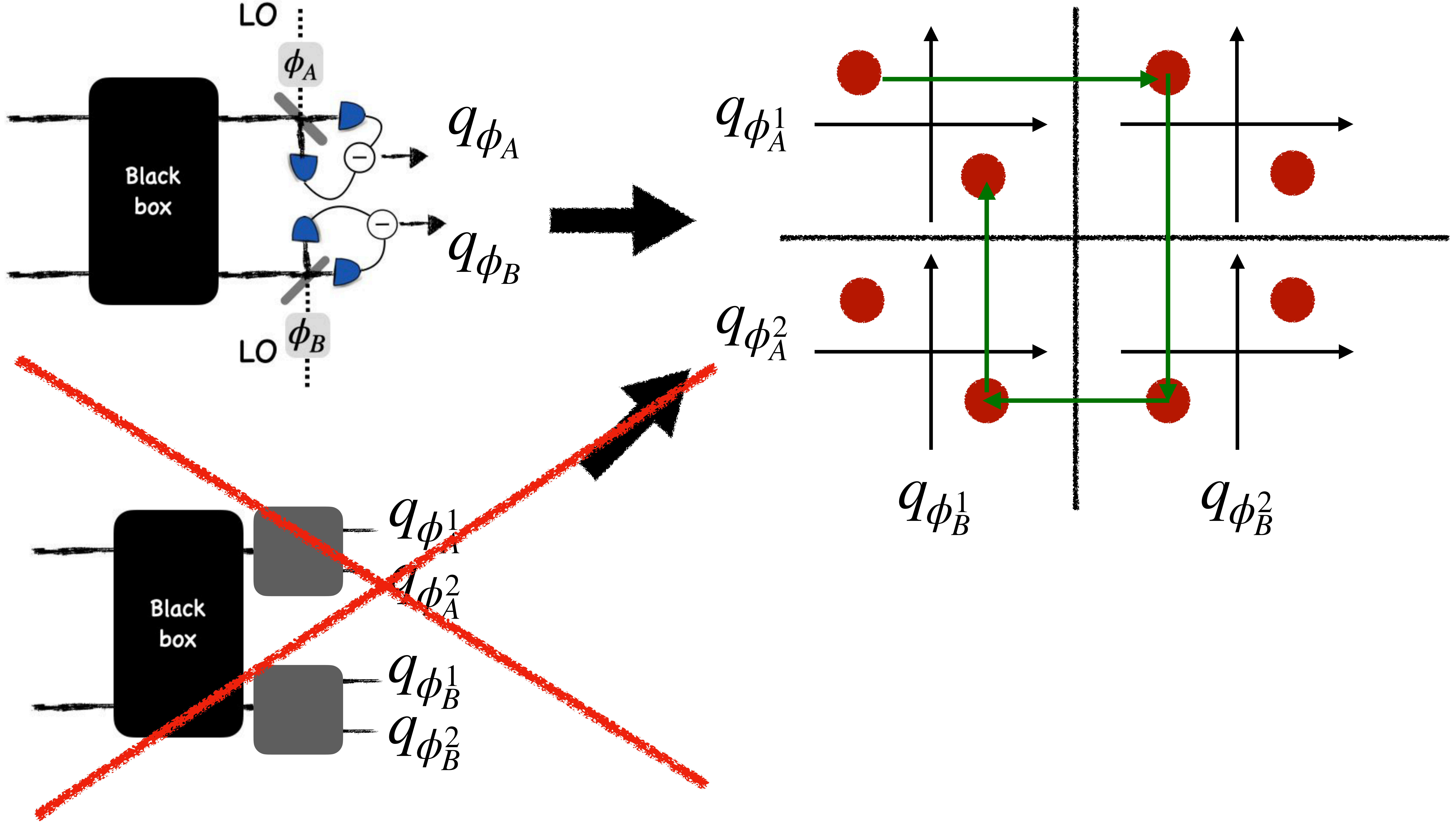
Non contextual if there is a global probability distribution that explains all the different contexts.



Measurement scenario
Empirical model
LO
ϕ_A
Black box
q_{ϕ_A}
q_{ϕ_B}
ϕ_B
LO
?
$q_{\phi_A^1}$
$q_{\phi_A^2}$
$q_{\phi_B^1}$
$q_{\phi_B^2}$
Black box
$q_{\phi_A^1}$
$q_{\phi_A^2}$
$q_{\phi_B^1}$
$q_{\phi_B^2}$

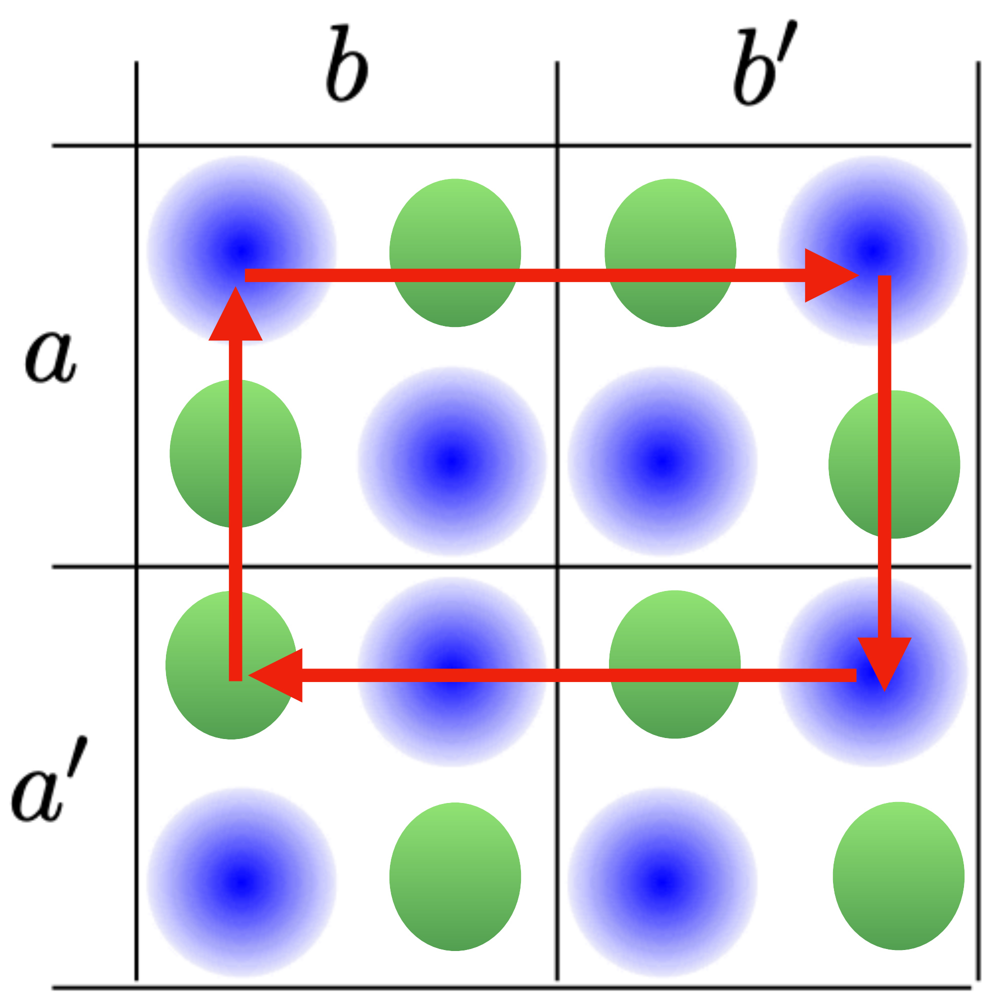
Measurement scenario

Empirical model



Quantifying contextuality

Intuition: what fraction of our measurement results are explained by a global probability distribution.



Intermediate behaviors are possible

non Contextual fraction > 0

Quantifying contextuality

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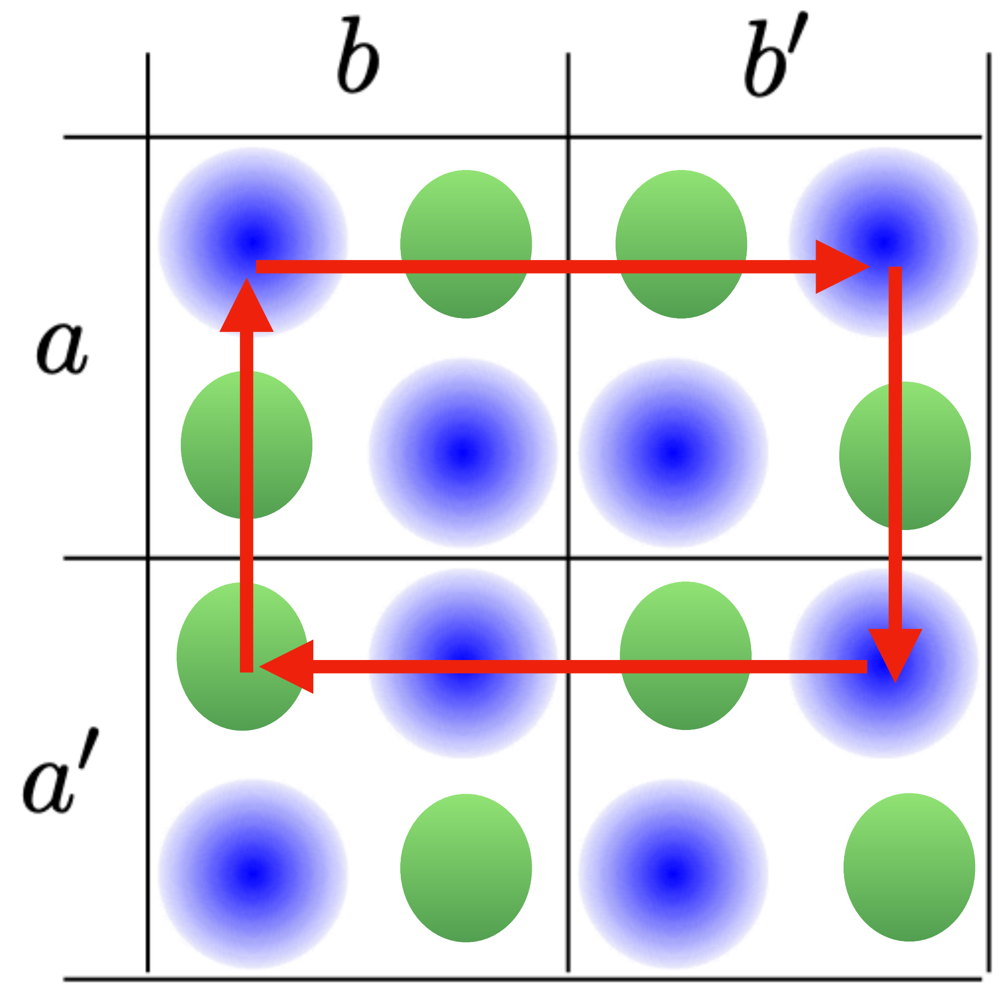

Given empirical behavior its non contextual fraction is given by

$$\sup_{\mu \geq 0} \left(\int_{O_X} d\vec{x}\mu(a, a^{'}, b, b^{'}) \qquad | \forall C \in M : \mu_C(x_C^A, x_C^B) \leq e_C(x_C^A, x_C^B) \right)$$

Infinite Linear program:

Primal

$$
\left\{
\begin{aligned}
&\text{Find } \mu \in \mathbb{M}_{\pm}(\boldsymbol{O}_X) \\
&\text{maximising } \mu(O_X) \\
&\text{subject to:} \\
&\qquad \forall C \in \mathcal{M},\ \mu|_C \ \leq\ e_C \\
&\qquad \mu \ \geq\ 0\ .
\end{aligned}
\right.
$$

Dual

$$
\left\{
\begin{aligned}
&\text{Find } (f_C)_{C\in\mathcal{M}} \in \prod_{C\in\mathcal{M}} C(O_C) \\
&\text{minimising } \sum_{C\in\mathcal{M}} \int_{O_C} f_C \ \mathrm{d}\, e_C \\
&\text{subject to:} \\
&\qquad \sum_{C\in\mathcal{M}} f_C \circ \rho^X_C \ \geq\ \mathbf{1}_{O_X} \\
&\qquad \forall C \in \mathcal{M},\ f_C \ \geq\ \mathbf{0}_{O_C}\ .
\end{aligned}
\right.
$$

Relaxation of the infinite linear program

1- Moment based SDP hierarchy

Relaxation of the infinite linear program

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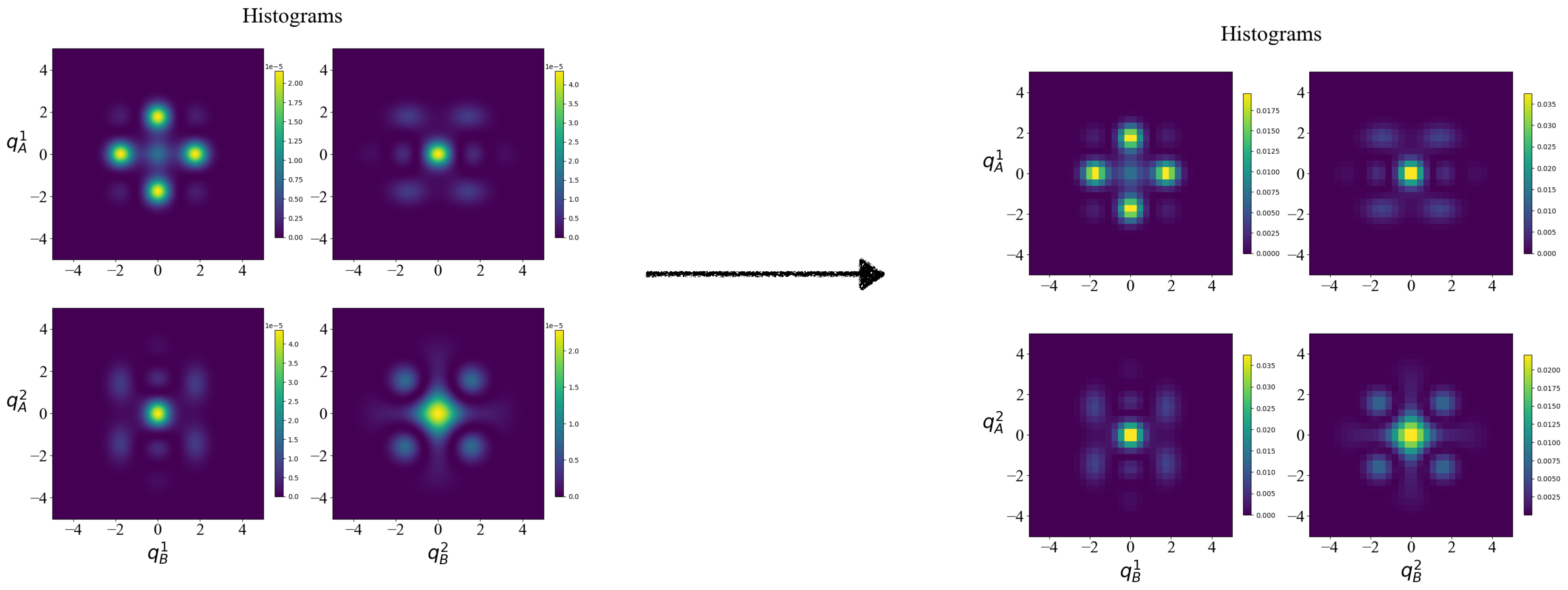
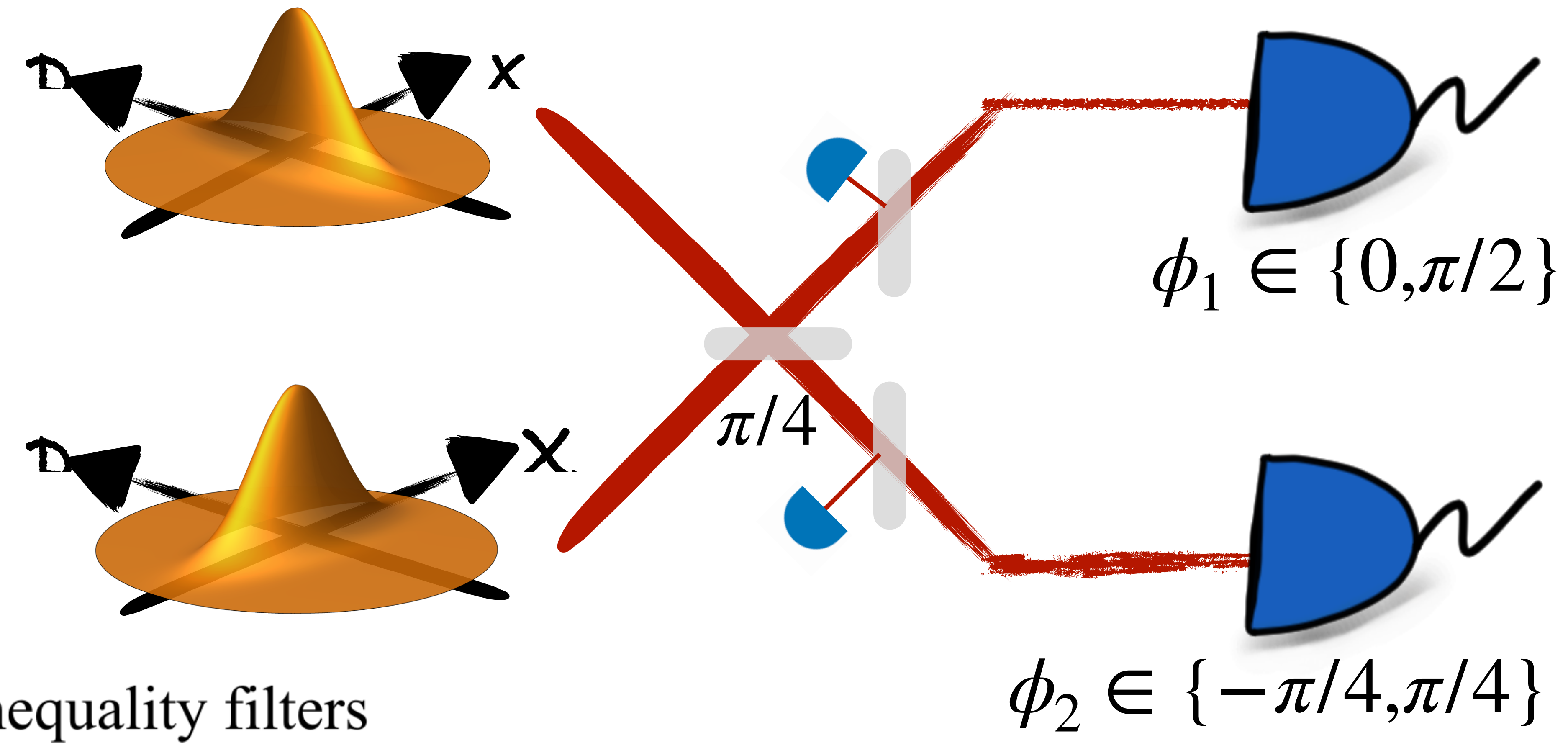
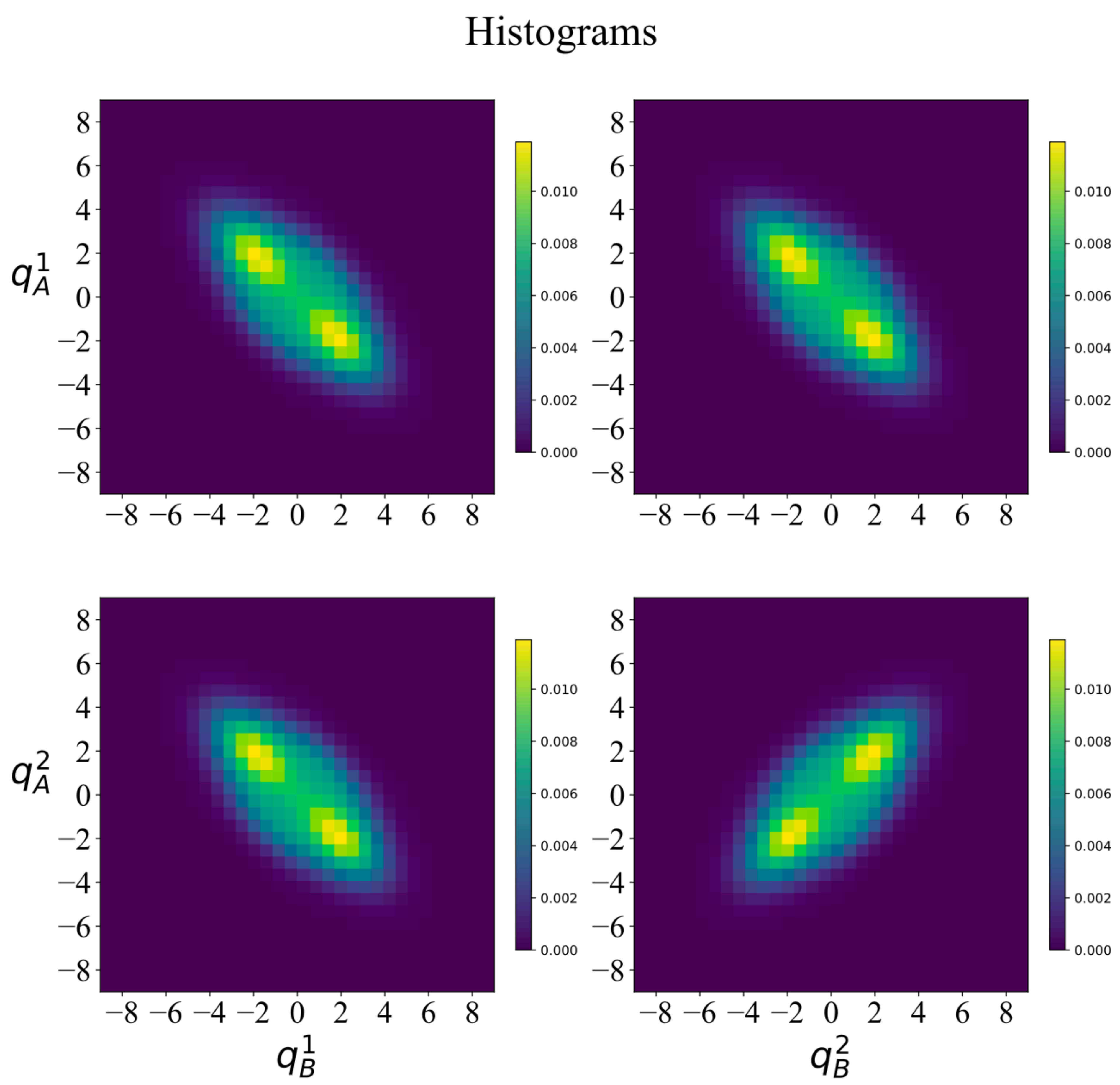
2- Finite Linear programs based on binning of the histograms

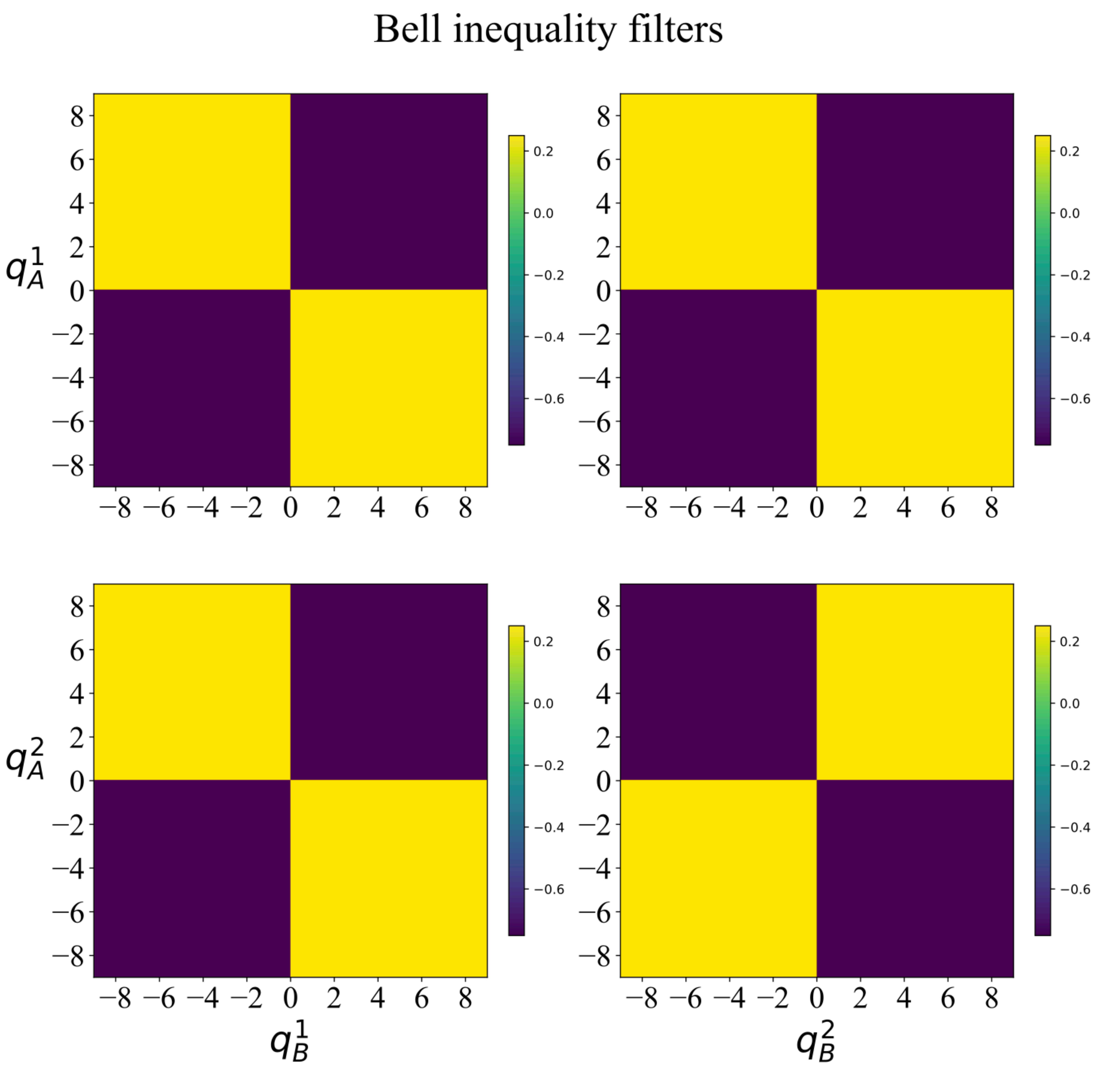


Two photon subtracted state

$r_1 = -r_2 = 0.68(5.9dB)$ $\quad CF = 2.37\ \%$



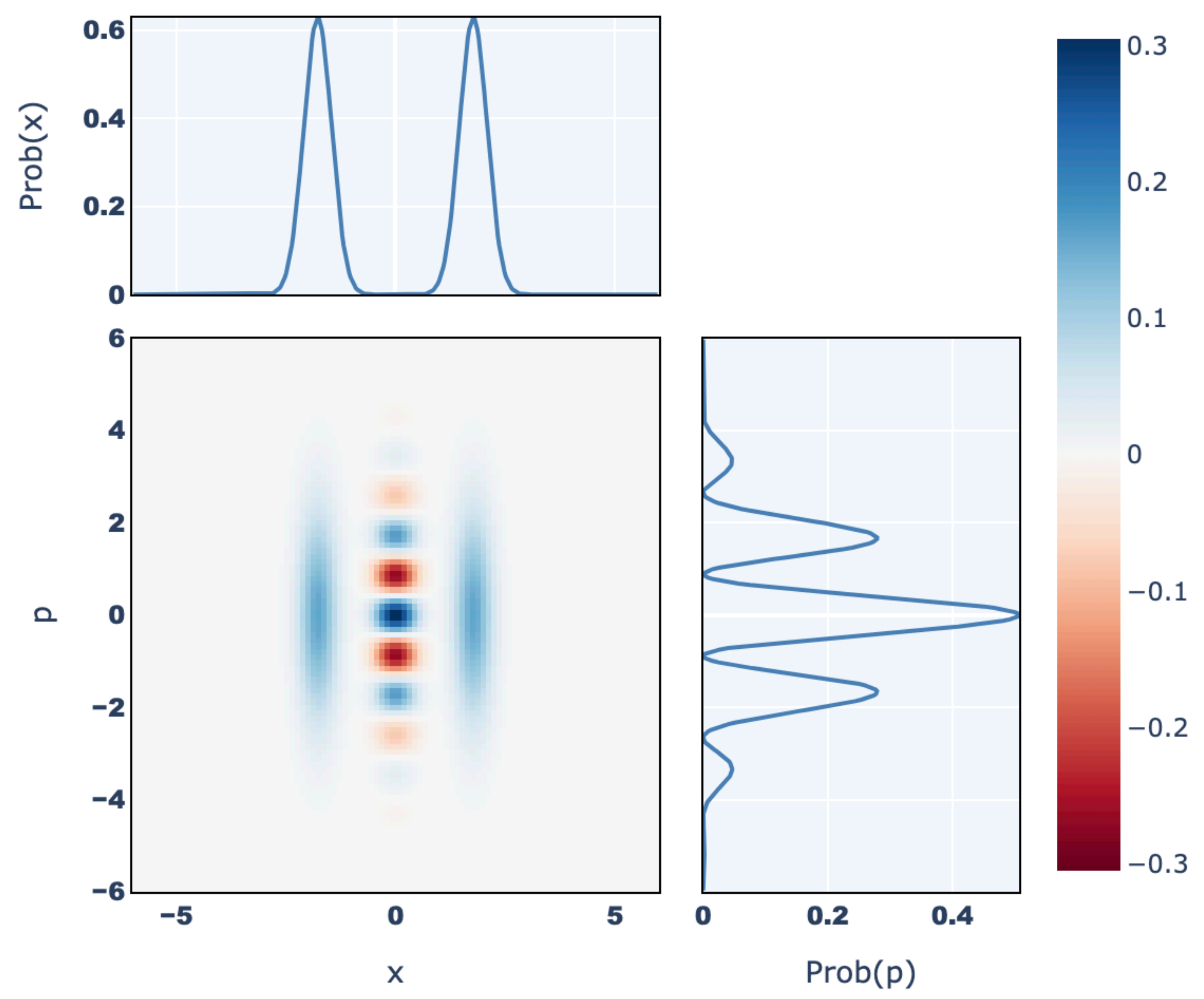


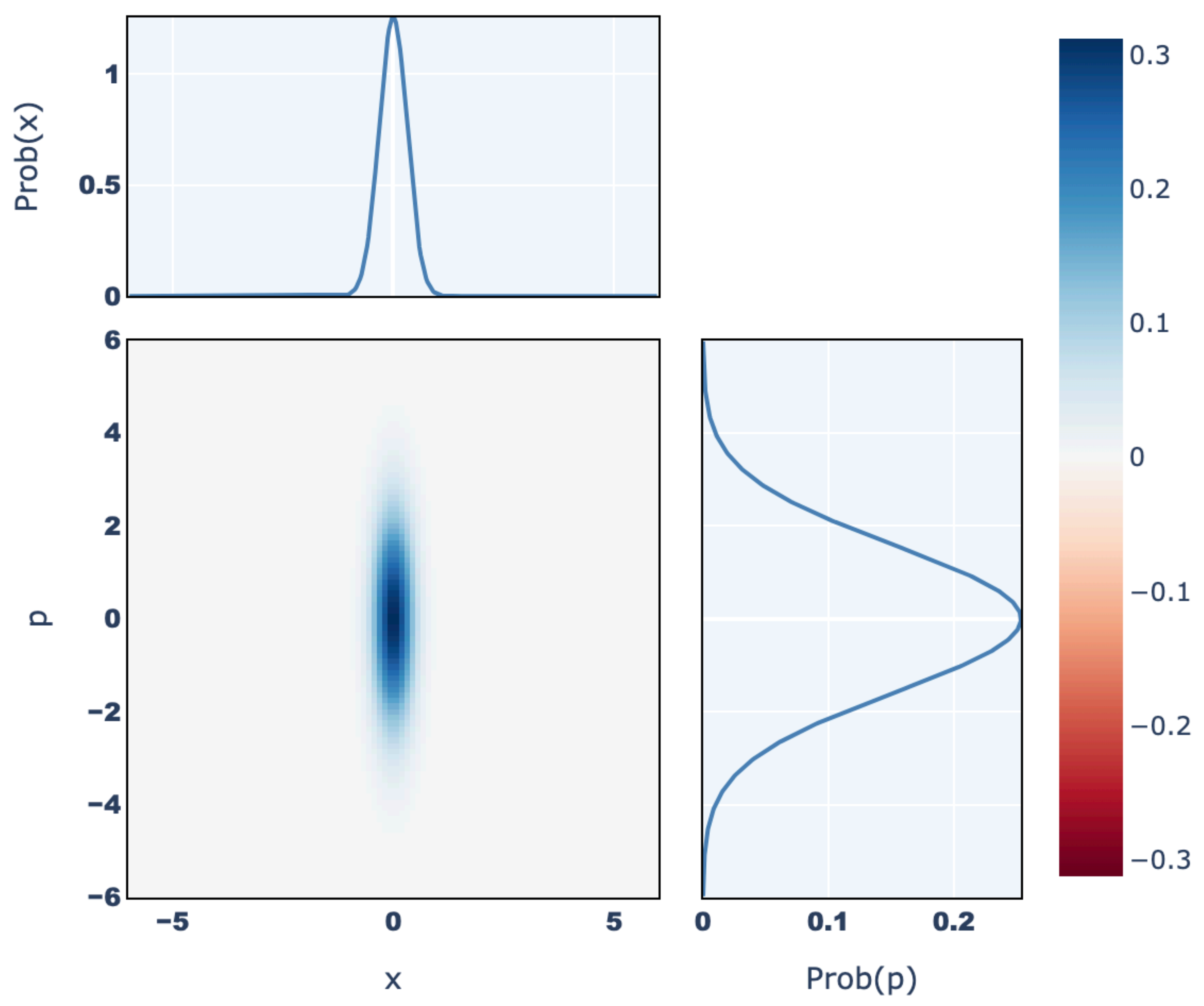


«GKP» entangled state

$$|\psi\rangle = \frac{\Big(|11\rangle - |00\rangle - (1+\sqrt{2})(|01\rangle + |10\rangle)\Big)}{2\sqrt{2+\sqrt{2}}}$$

$$|0> = \frac{|\alpha, r\rangle + |-\alpha, r\rangle}{\sqrt{2}} \qquad |1> = \hat{S}(r)|0\rangle$$





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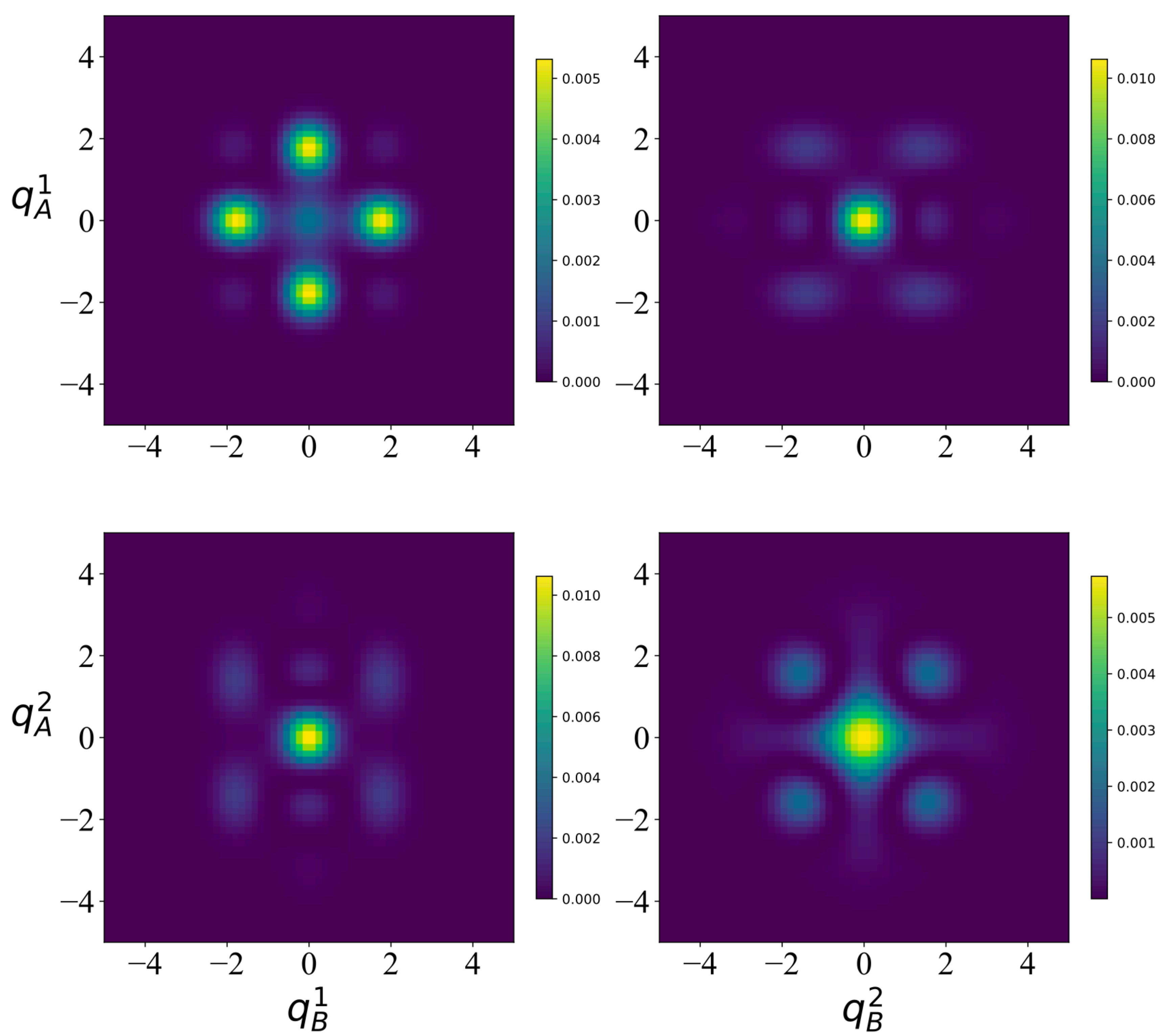
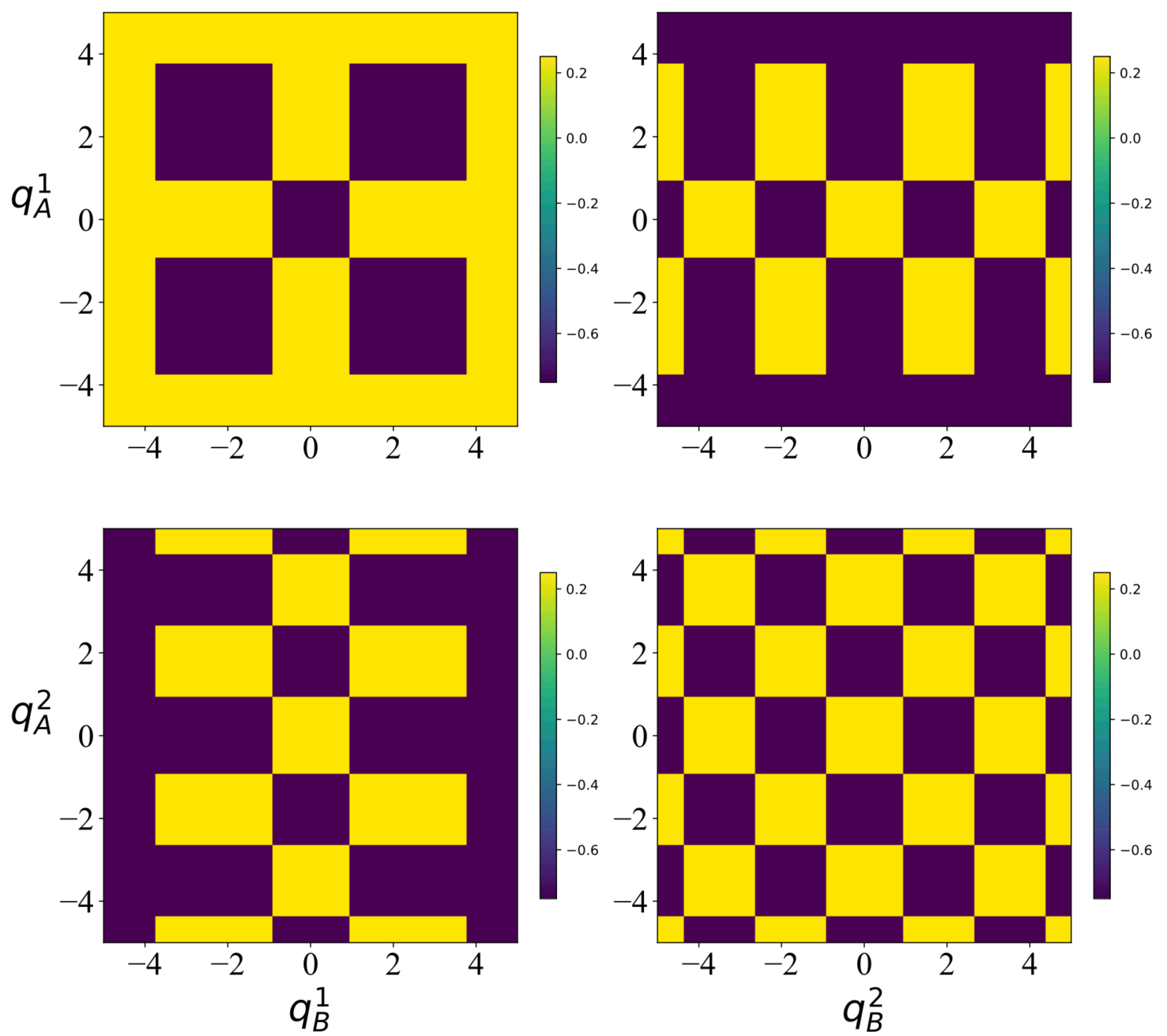
$$|1> = \frac{|\alpha, r\rangle + |-\alpha, r\rangle}{\sqrt{2}} \qquad |1> = \hat{S}(r)|0\rangle$$

$$CF = 26.2\,\%$$

Histograms

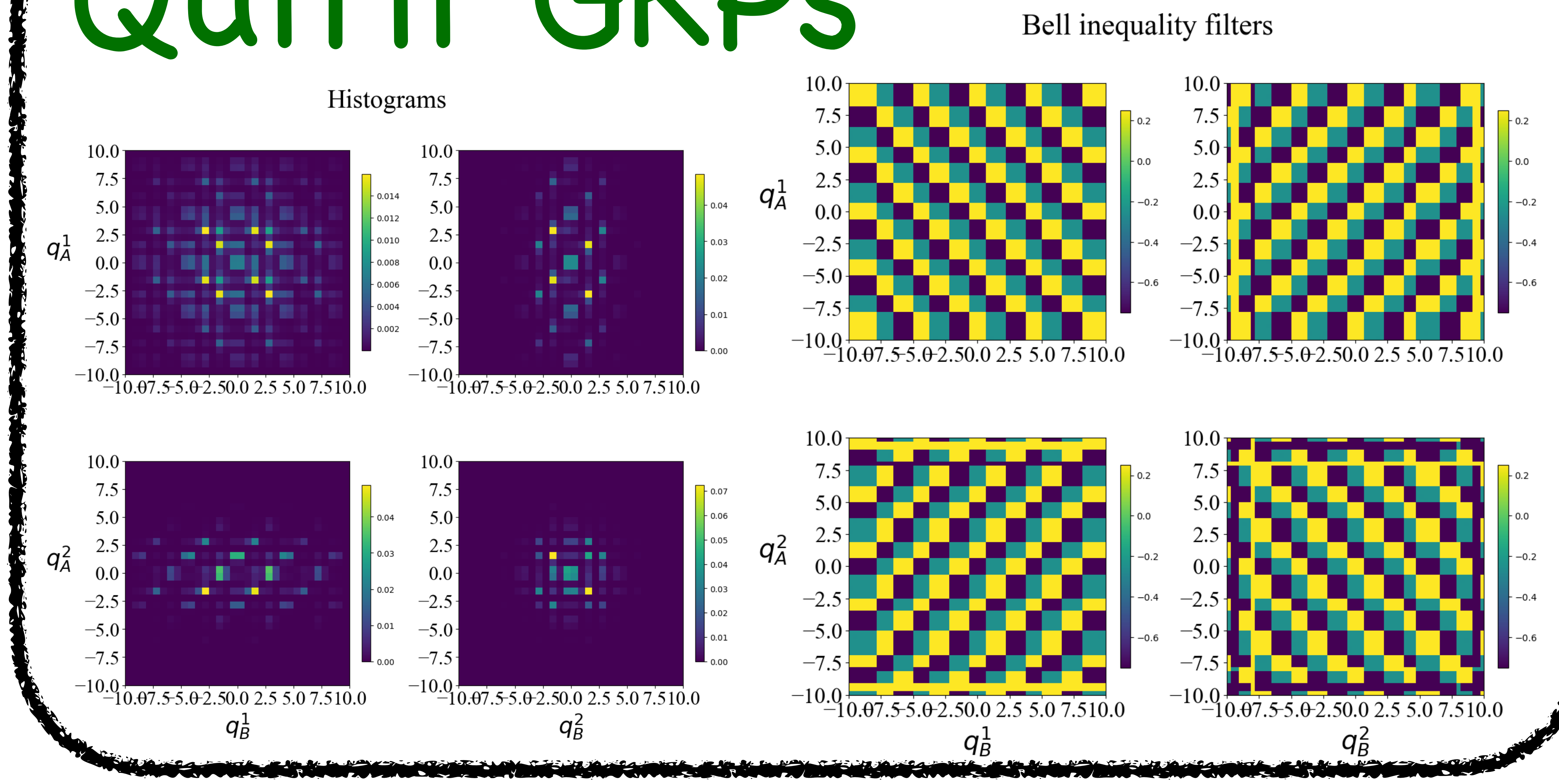


Bell inequality filters



Qutrit GKPs

CF: 26 %



Hybrid systems

$$|\psi\rangle = \frac{|0\rangle_{DV}|0\rangle_{CV} + |1\rangle_{DV}|1\rangle_{CV}}{\sqrt{2}}$$

CV encoding: GKP, even vs odd cats ...

Multi-mode systems

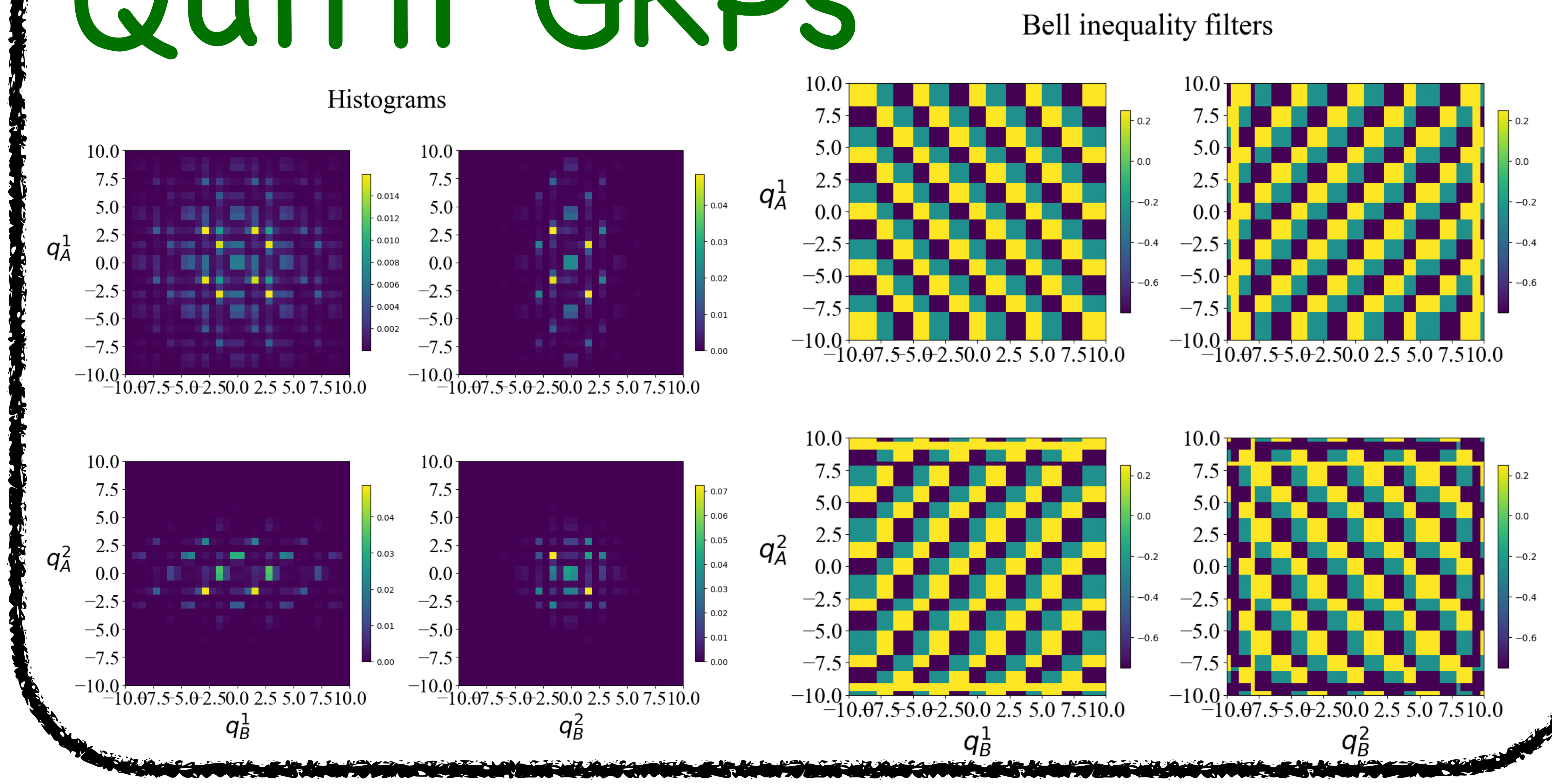
$$|\psi\rangle = \frac{|000\rangle_{CV} + |111\rangle_{CV}}{\sqrt{2}}$$

Encoding: GKP (low quality ones)

CF: 46 % !!! (well beyond CHSH)

Qutrit GKPs

CF: 26 %



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Encoding: GKP (low quality ones)

CF: 46 % !!! (well beyond CHSH)

Many open questions:

- moment based inequalities do not exist at low order
- finding good states is a daunting task

Thank you!

2- Discrete linear program

Infinite LP (primal)

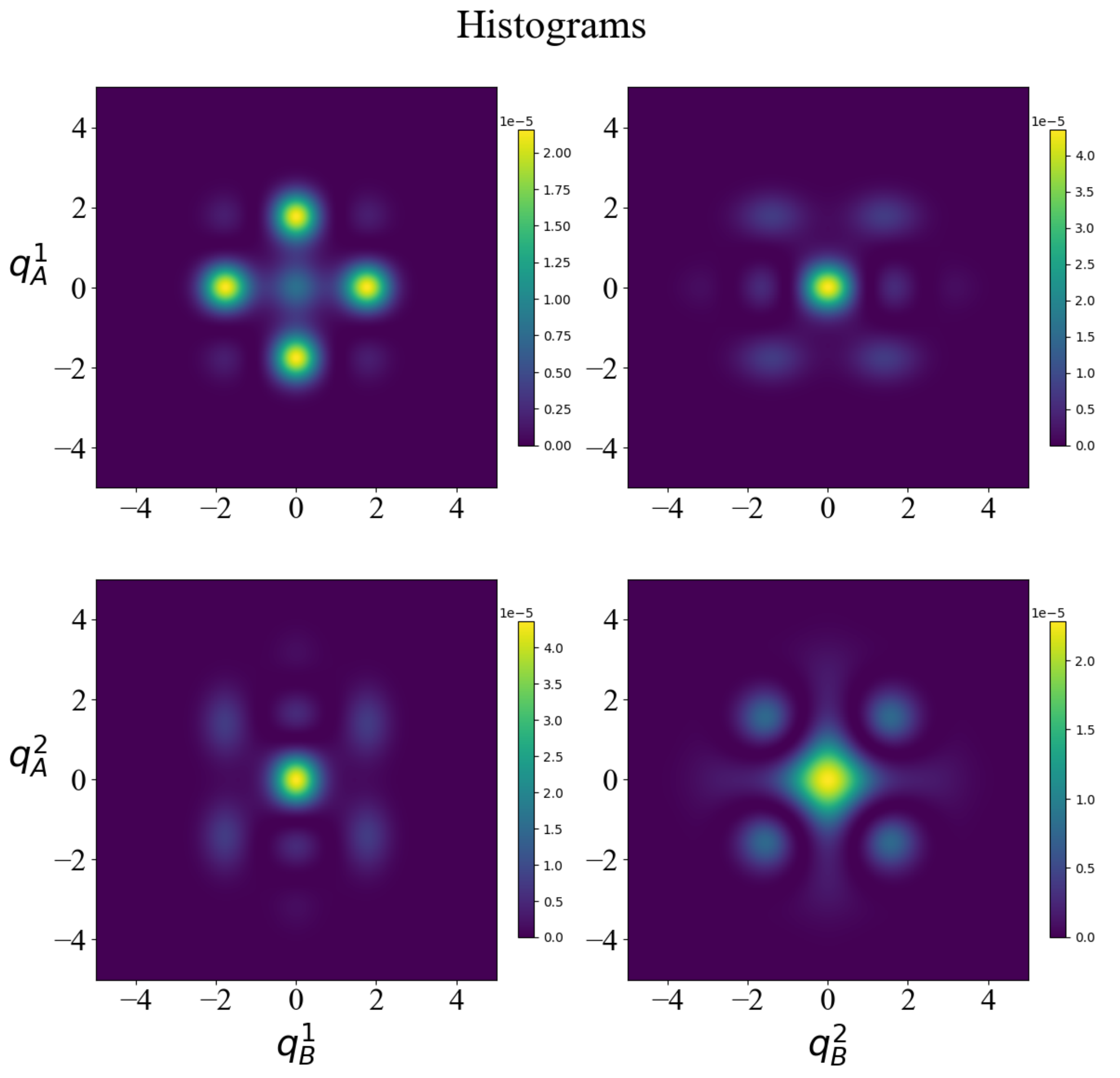
$$
(P)\begin{cases}
\text{Find} & \mu \in \mathrm{M}_{\pm}((\mathrm{O}_X)) \\
\max & \mu(\mathrm{O}_X) \\
\text{s.t.} & \forall C \in \mathscr{M} \mu|_C \leq e_C \\
 & \mu \geq 0.
\end{cases}
$$

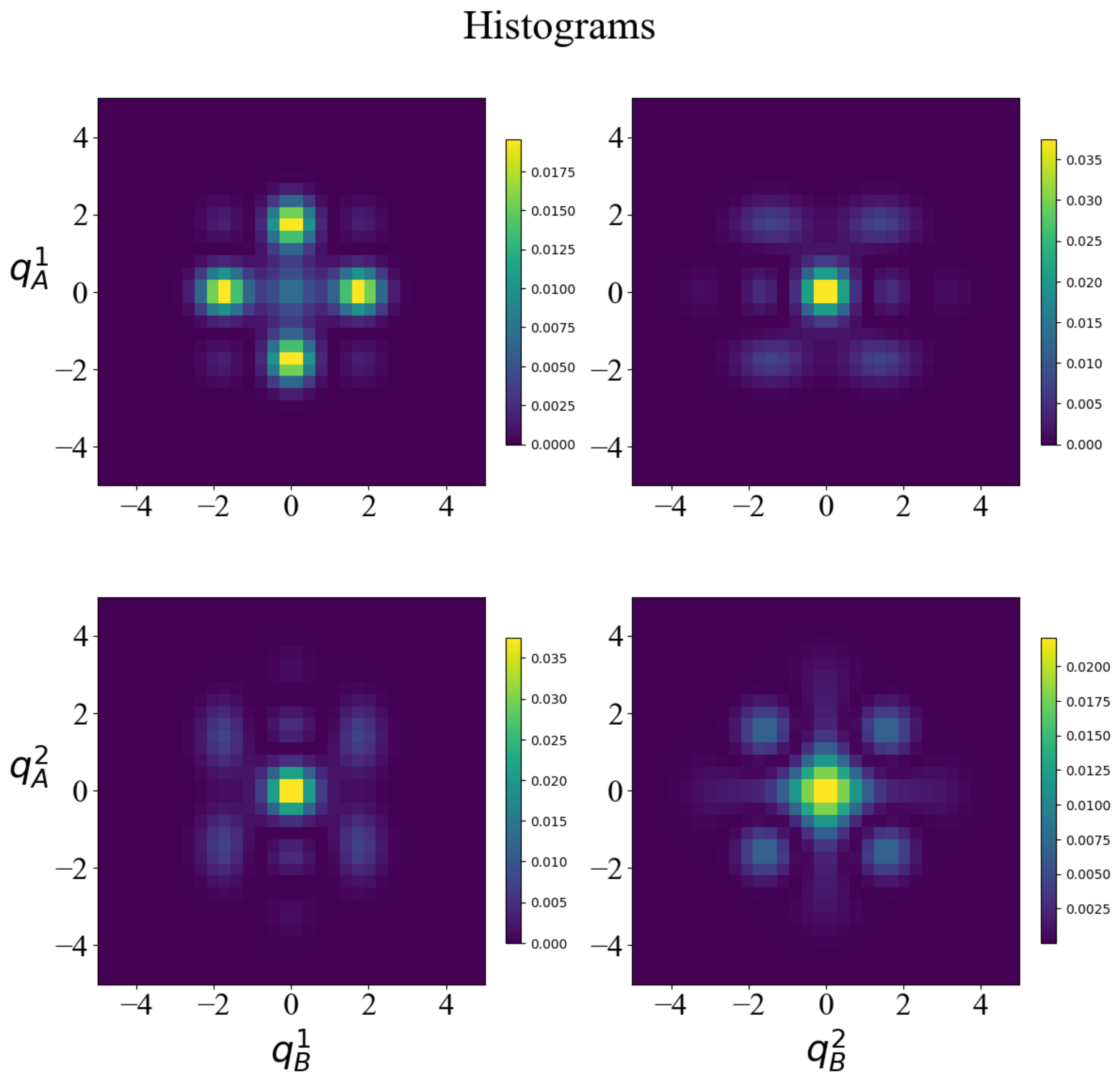
Discretization

$O_X = \mathbb{R} \rightarrow \{-A, -A+\delta, \ldots, A-\delta, A\}$

N_{bins}

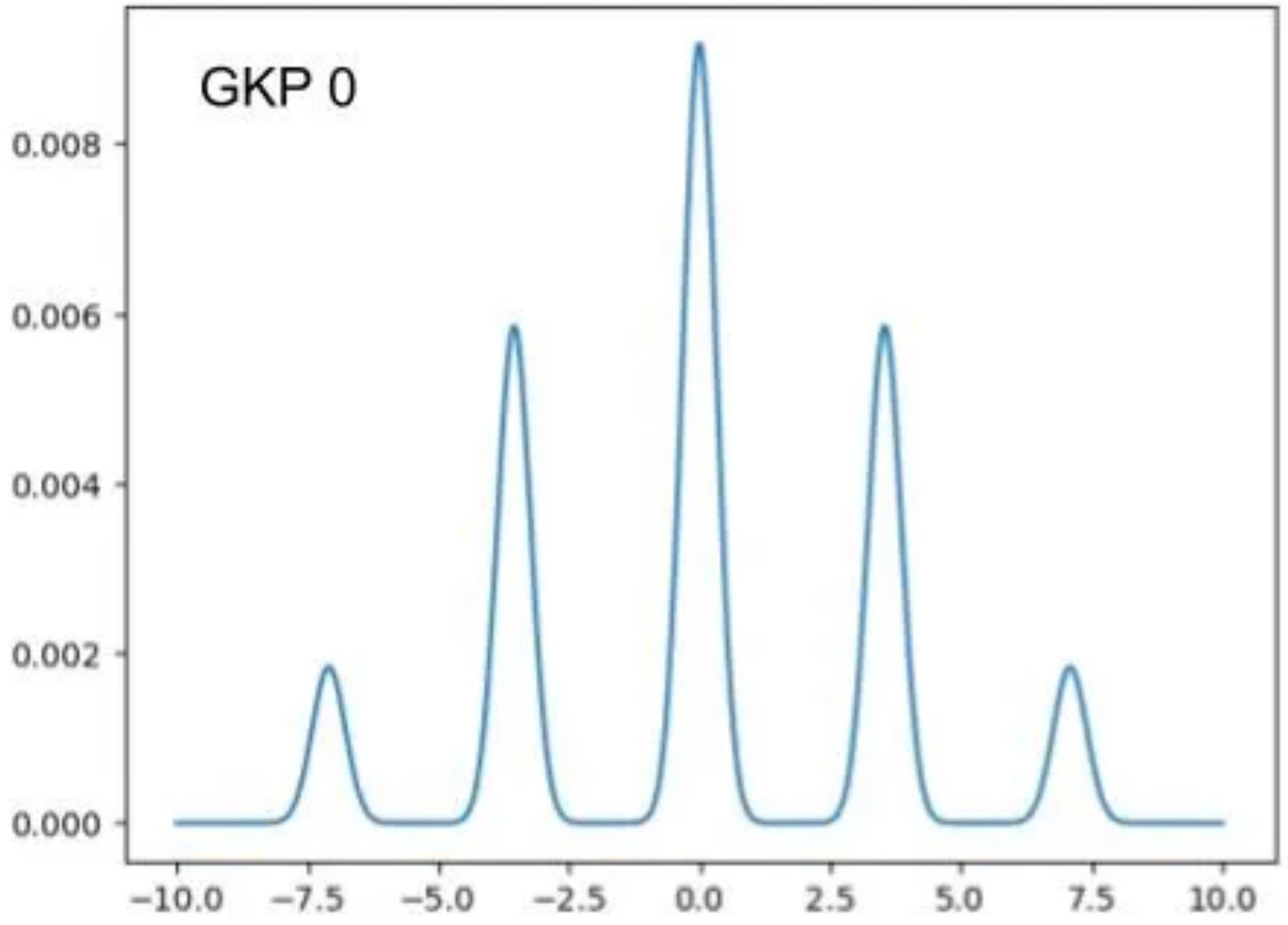
$$
(P,D)\begin{cases}
\text{Find} & \mu \in \mathbb{M}(N_{bins}^{|X|}) \\
\max & \sum_{i,j,k,l} \mu_{ijkl} \\
\text{s.t.} & \forall C \in \mathscr{M} \mu|_C \leq e_C \\
 & \mu \geq 0.
\end{cases}
$$

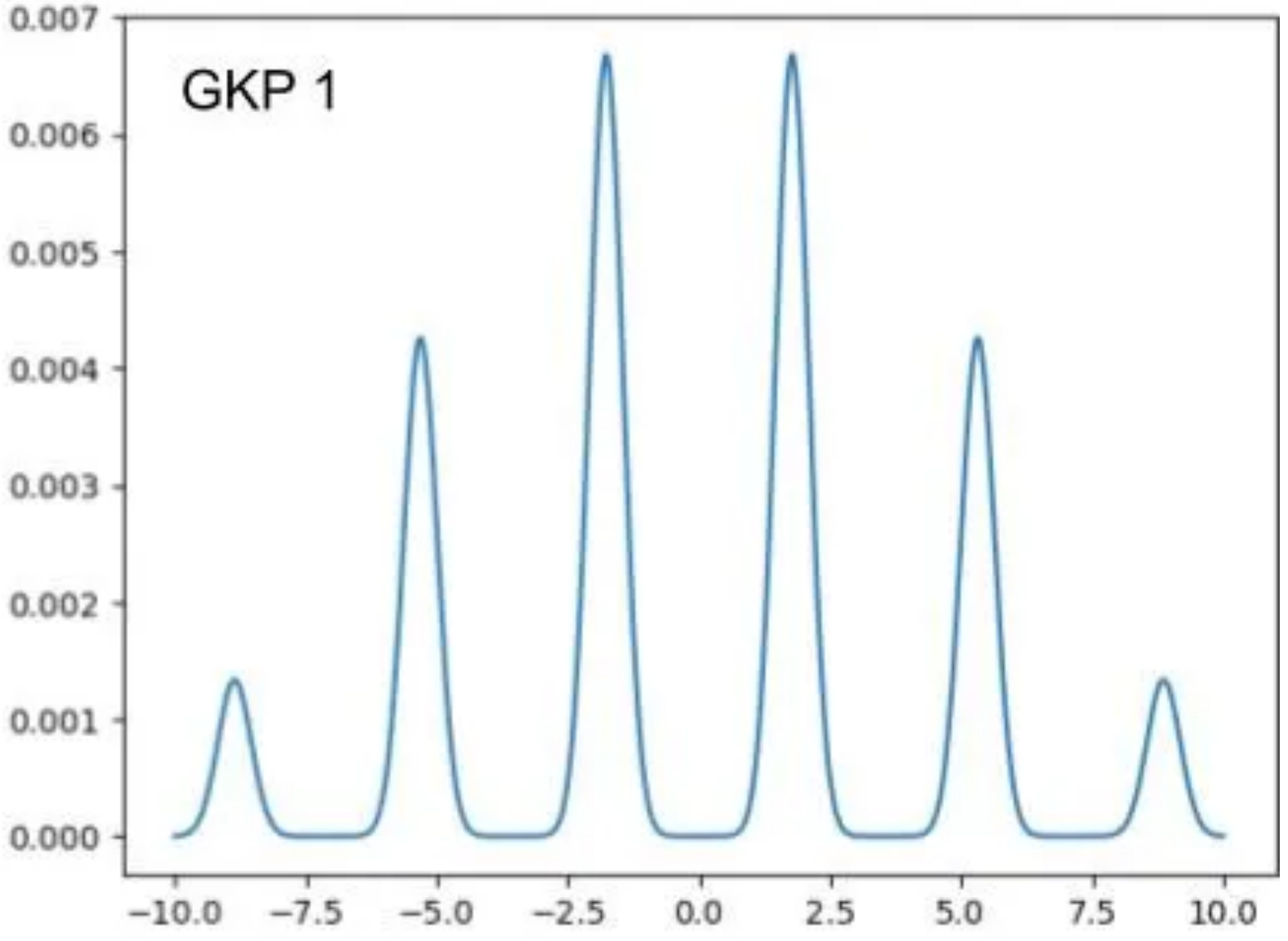




GKP entangled state

$$|\psi\rangle = \frac{\left(|11\rangle - |00\rangle - (1+\sqrt{2})(|01\rangle + |10\rangle)\right)}{2\sqrt{2+\sqrt{2}}}$$





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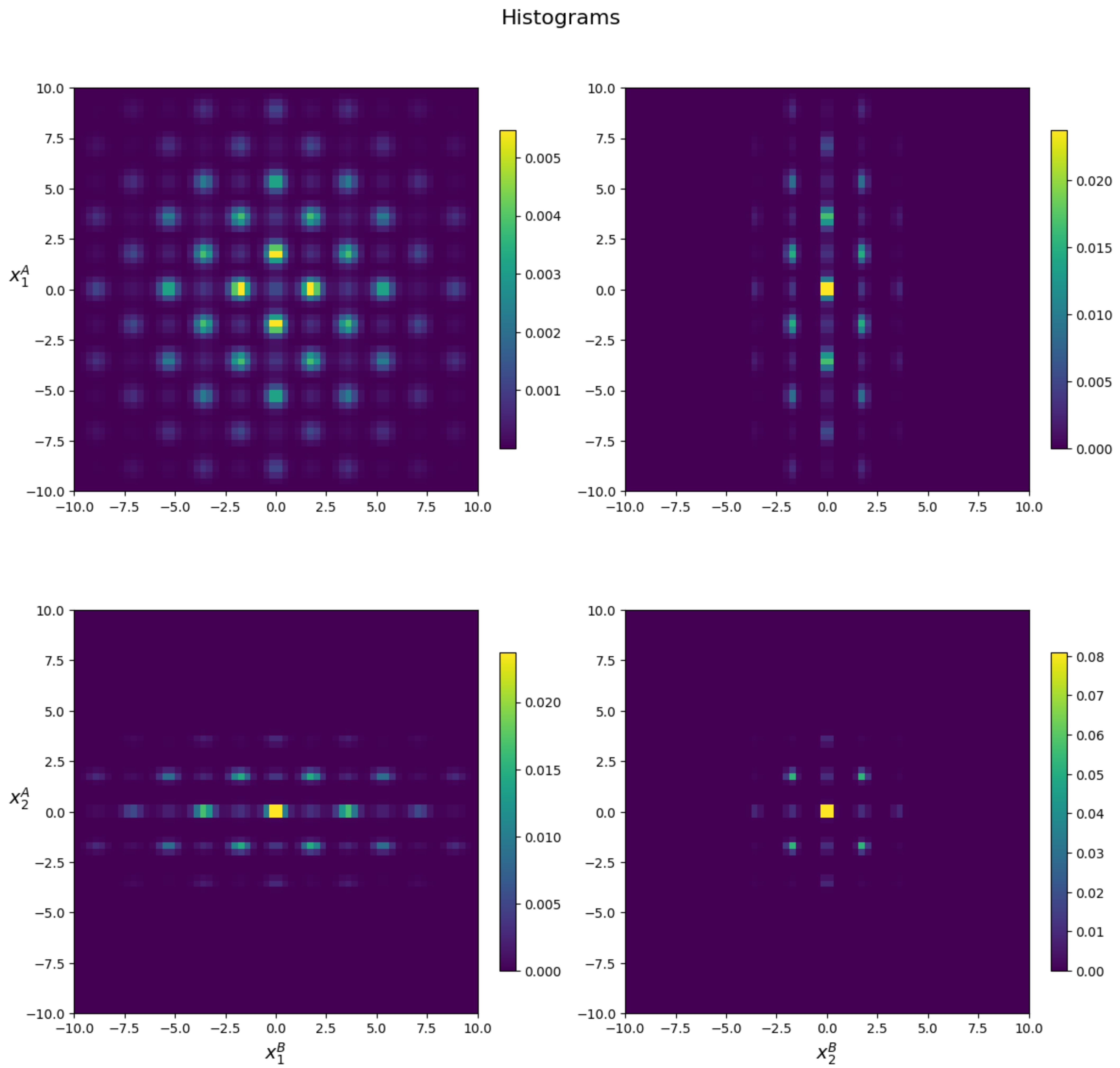
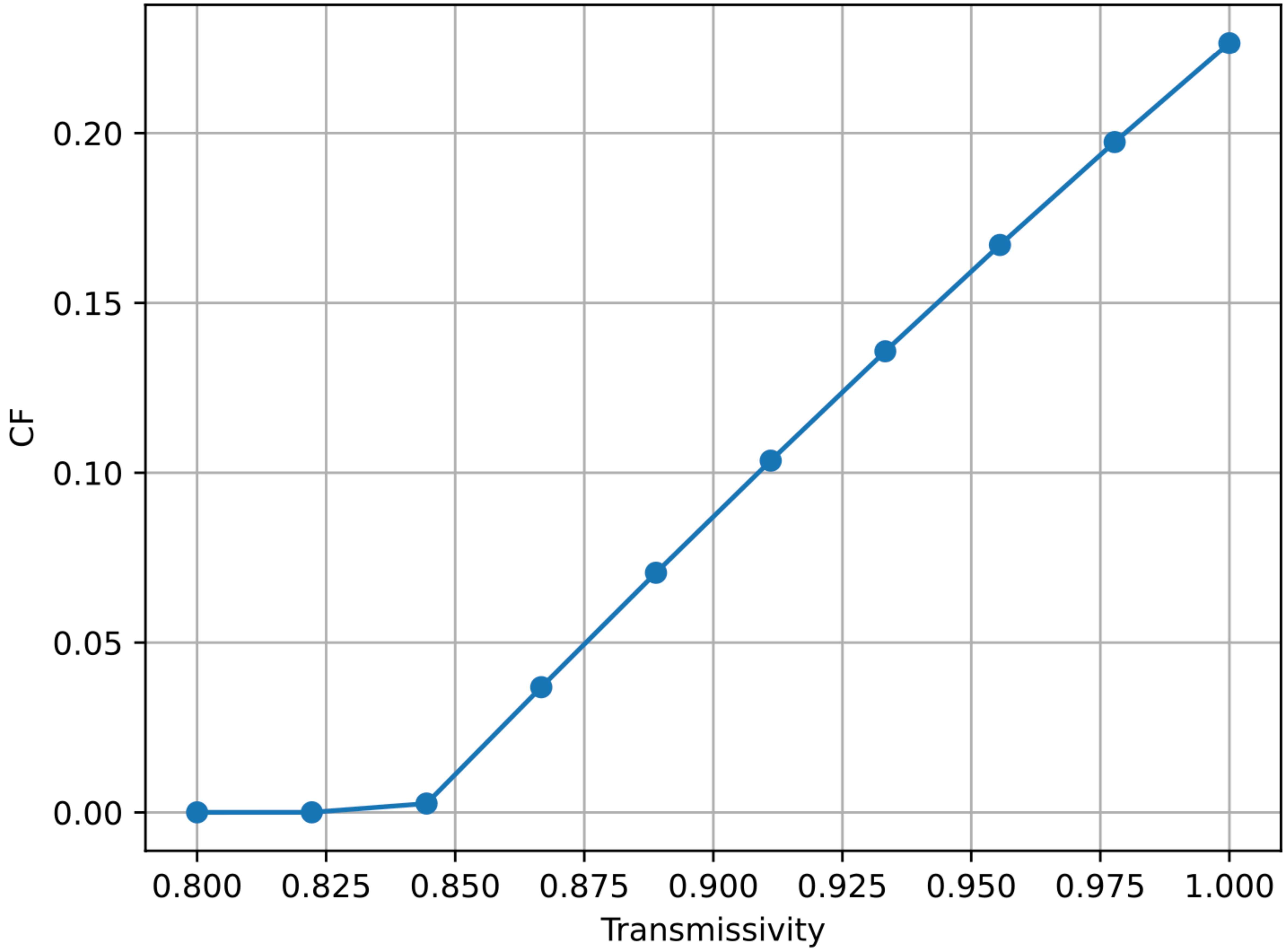


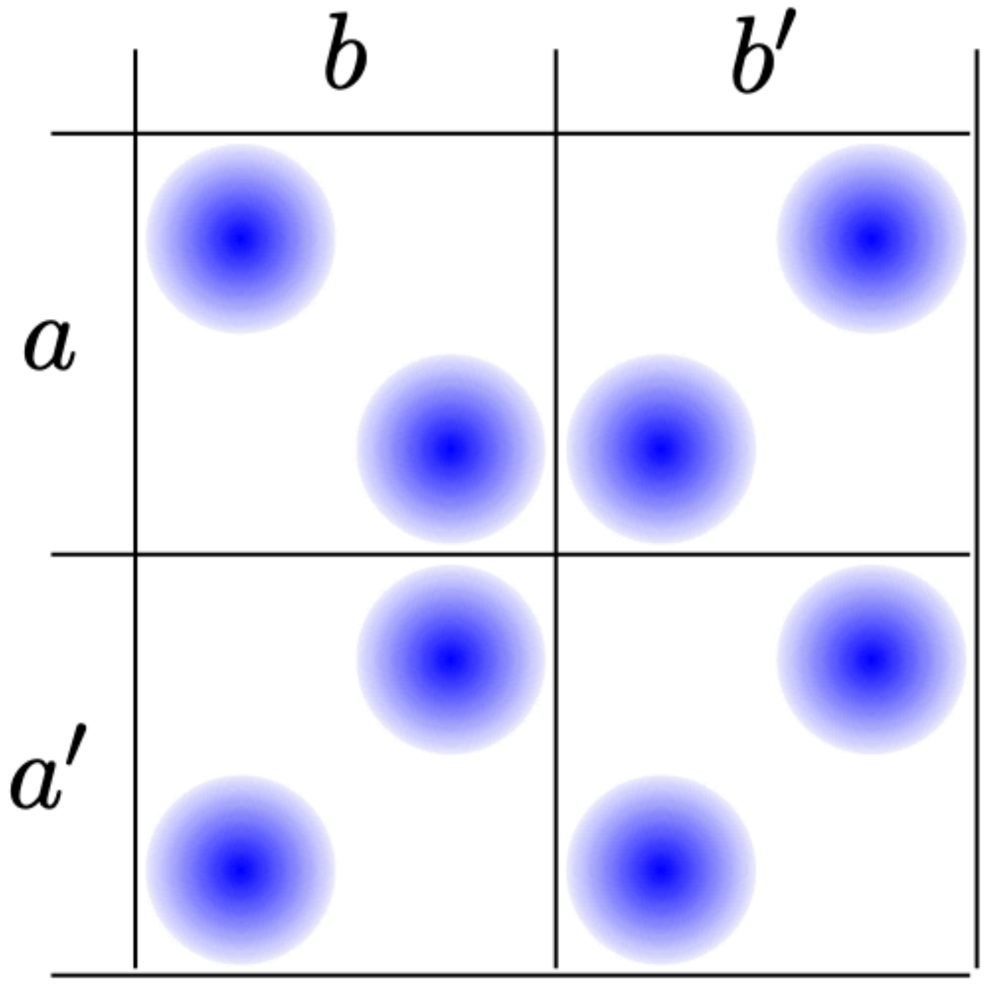
No. bins	8	16	32	64
CF	0	0.2434	0.3452	0.3685

Loss resilience



Generalized Bell inequality

$$\begin{cases} \text{Find } (\beta_C)_{C\in\mathcal{M}} \in \prod_{C\in\mathcal{M}} C(O_C) \\ \text{maximising } \sum_{C\in\mathcal{M}} \int_{O_C} \beta_C \, \mathrm{d}\, e_C \\ \text{subject to:} \\ \qquad \sum_{C\in\mathcal{M}} \beta_C \circ \rho_C^X \leq \mathbf{0}_{O_X} \\ \qquad \forall C \in \mathcal{M},\ \beta_C \leq |\mathcal{M}|^{-1} \mathbf{1}_{O_C}\ . \end{cases}$$



0.25	−0.75	−0.75	0.25
−0.75	0.25	0.25	−0.75
−0.75	0.25	−0.75	0.25
0.25	−0.75	0.25	−0.75

if no contextuality

$$e_C \to \mu_X \qquad \sum_{C\in\mathscr{M}} \int_{O_C} \beta_C de_C = \sum_{C\in\mathscr{M}} \int_{O_X} \beta_C \cdot \rho_C^X d\mu = \int_{O_X} d\mu \left(\sum_C \beta_C \cdot \rho_C^X \right) \leq 0$$