PERIMETER INSTITUTE

QUIDIQUA 3

Classical explainability and quasiprobability representations

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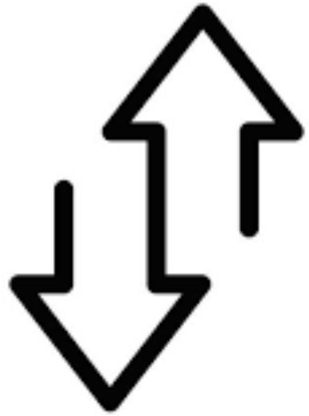
(see also Rafael Wagner's talk)

Quasiprobability representations



Quantum foundations

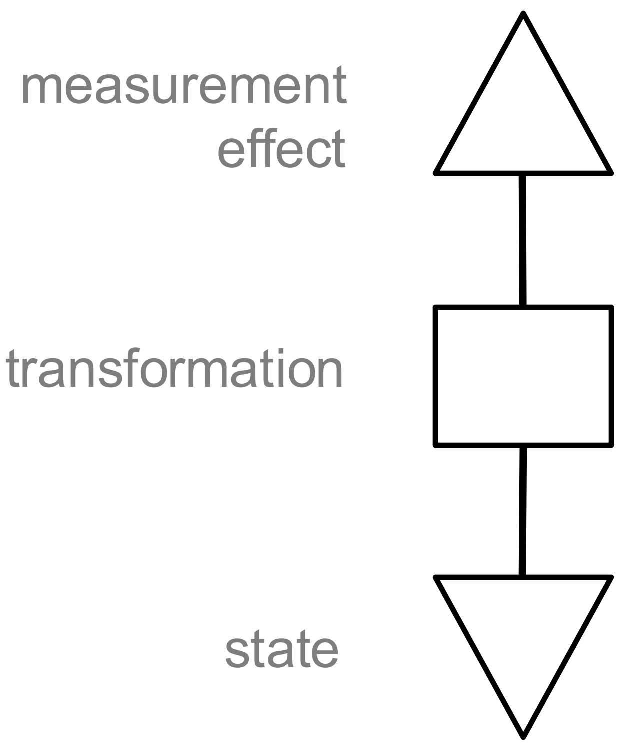
Nonclassicality

The things that we want to explain (classically or otherwise) are phenomena/experiments/theories

Generalized Probabilistic Theory

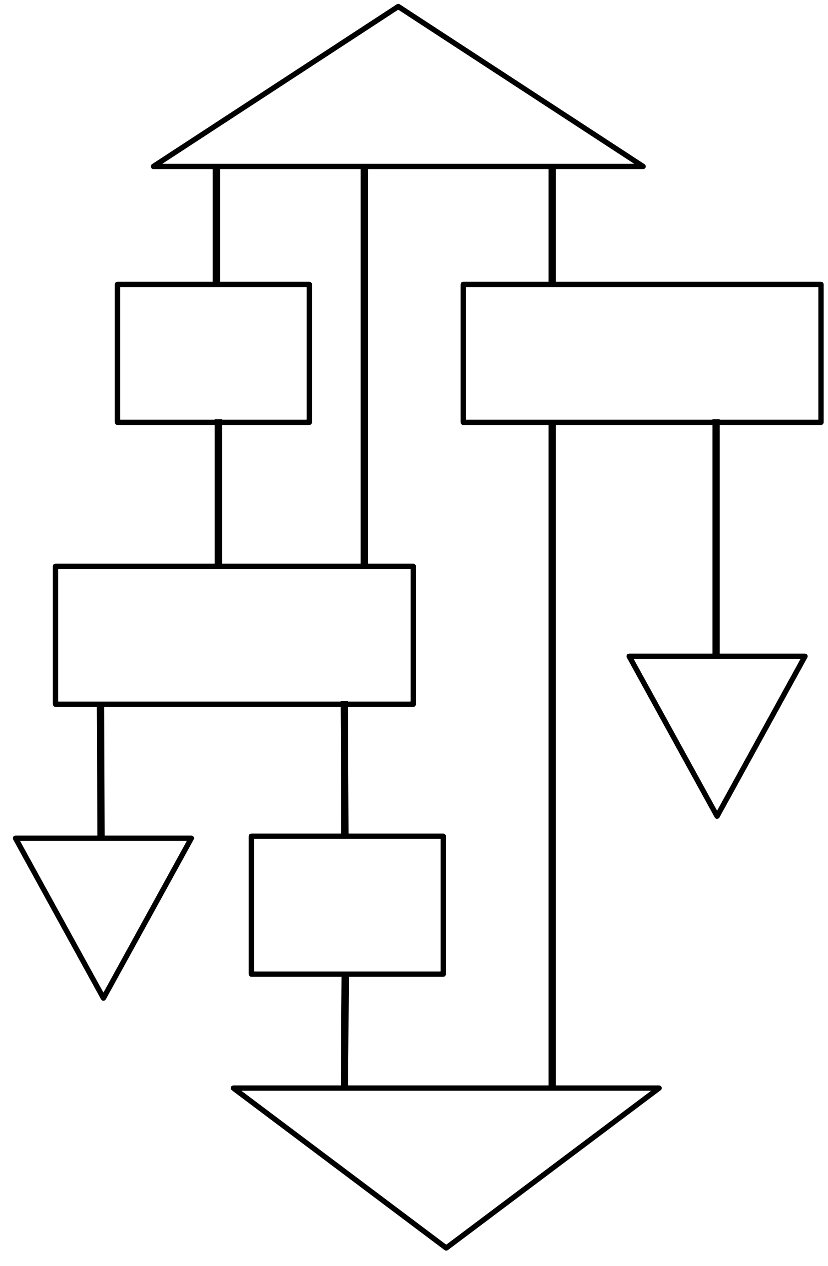
= a possible theory of the world

a collection of systems and a collection of processes
that can be composed together



= prob(effect|transformed state)

Can generate arbitrary circuits/experiments by composition:

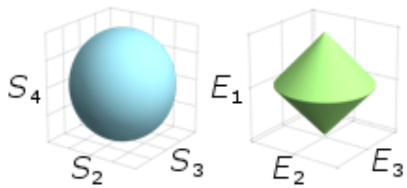


Different theories are defined by their:

1. Convex geometry

2. Compositional structure

qubit



Different theories are defined by their:

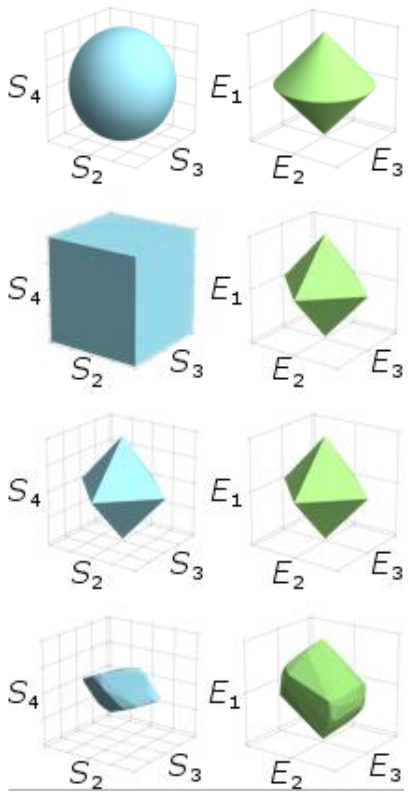
1. Convex geometry

qubit

Boxworld (3d)

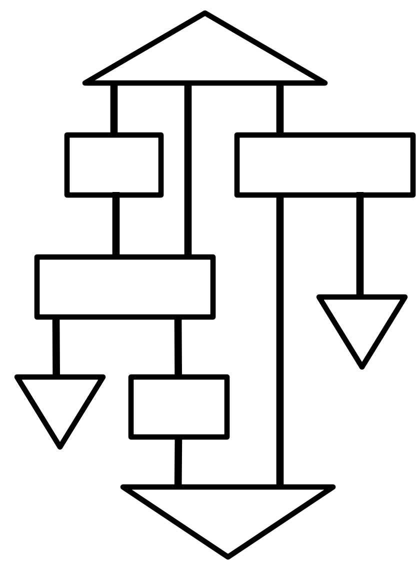
Spekkens toy theory

random GPT



2. Compositional structure

-probabilities = inner products
-$T_1(T_2)=T_3$
-multipartite states
-multipartite effects
-etc



describing a theory:

GPT

all possible systems, processes, and circuits

describing an experiment

GPT fragment

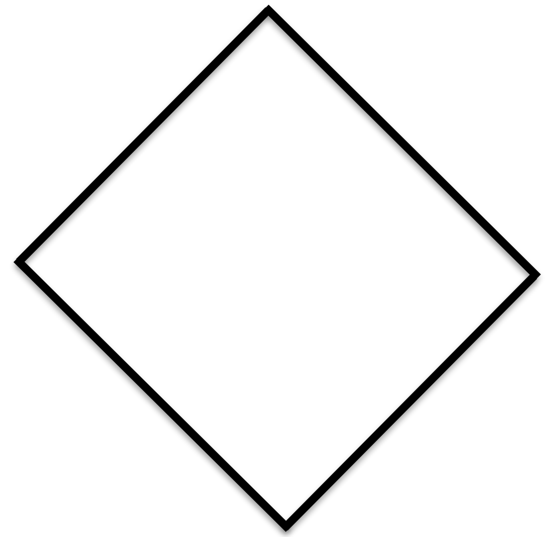
subset of systems, processes, and circuits

Classical explainability
(prepare-measure)

classical systems

states

effects

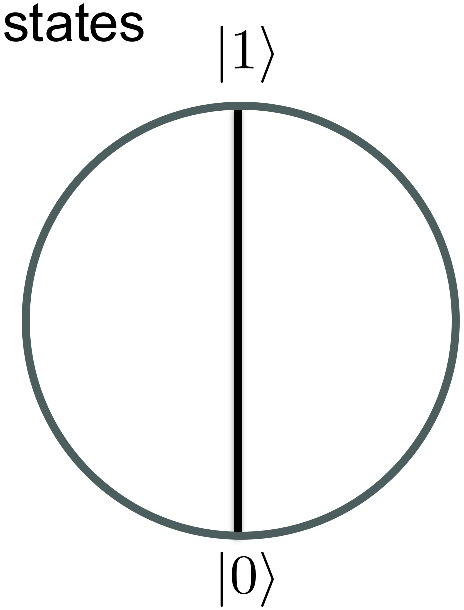


d = 2

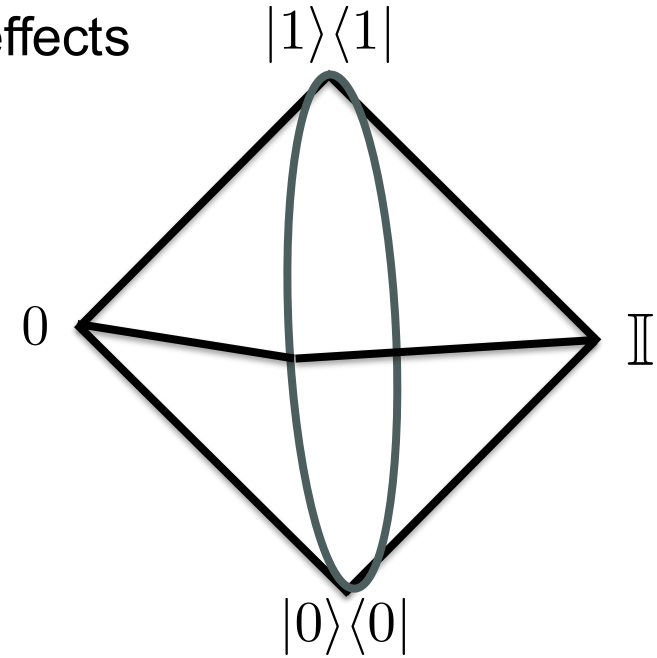
classical systems

$d = 2$

states



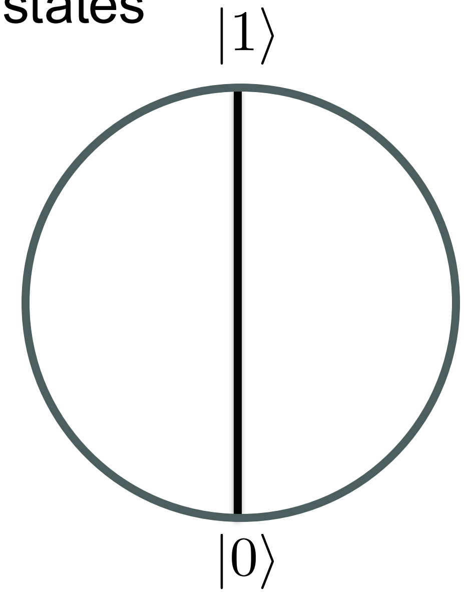
effects



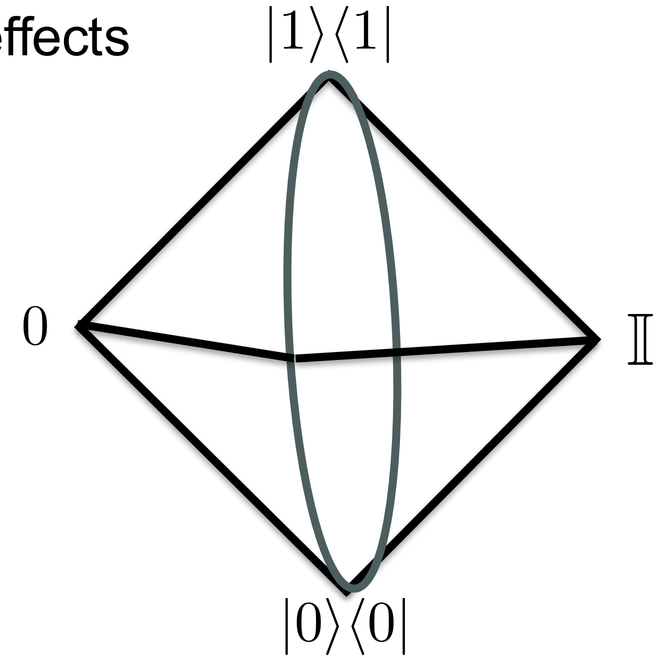
classical systems are consistent with (explainable by) quantum theory:

d = 2

states



effects



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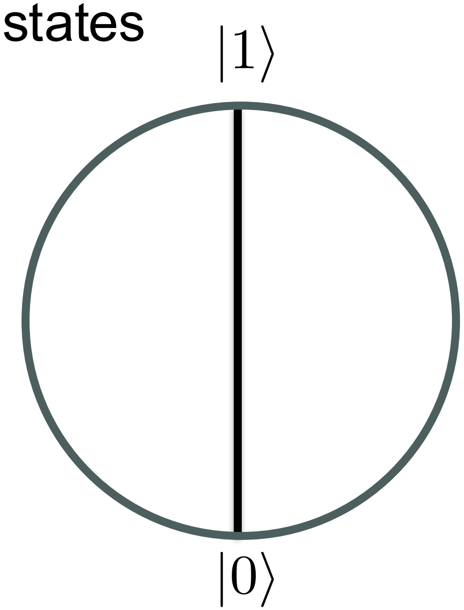
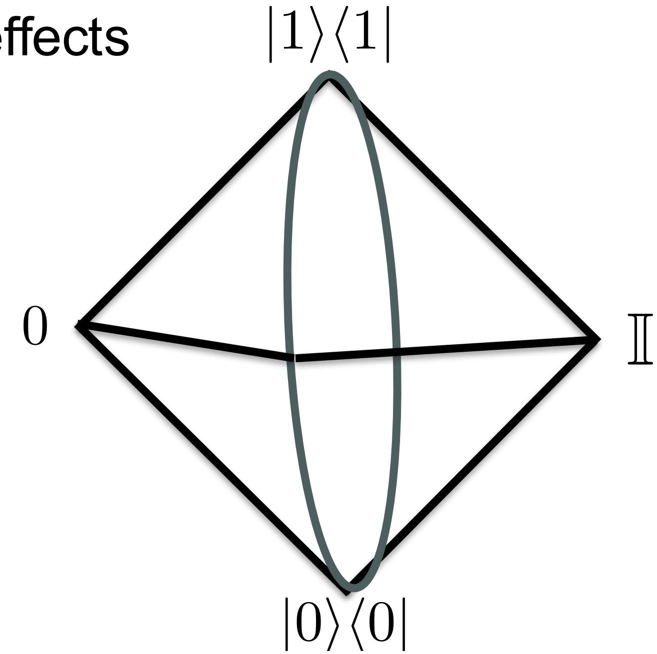
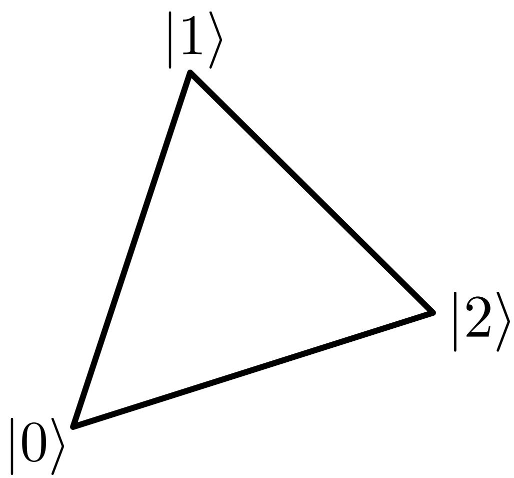
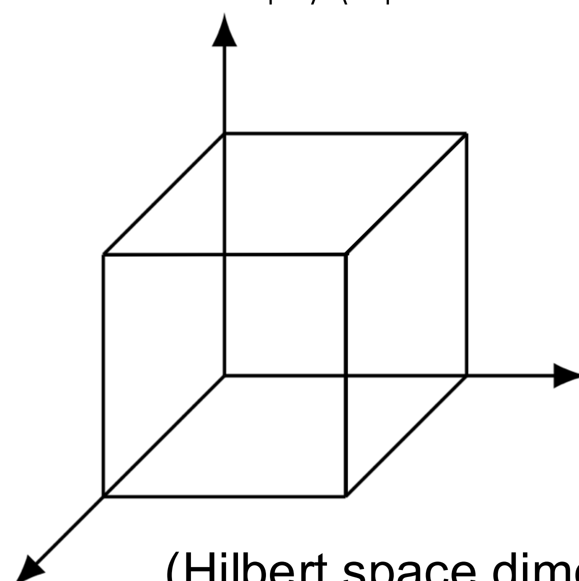




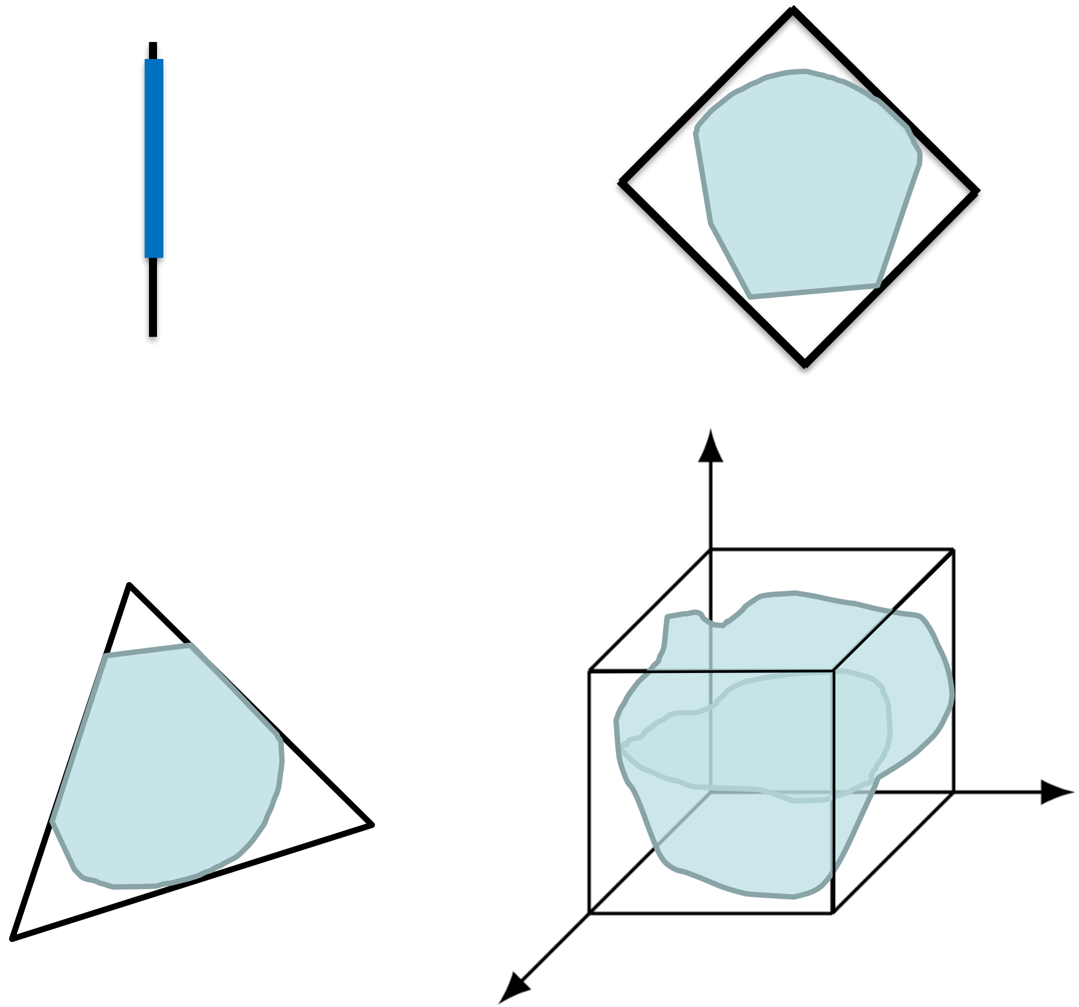
d = 3





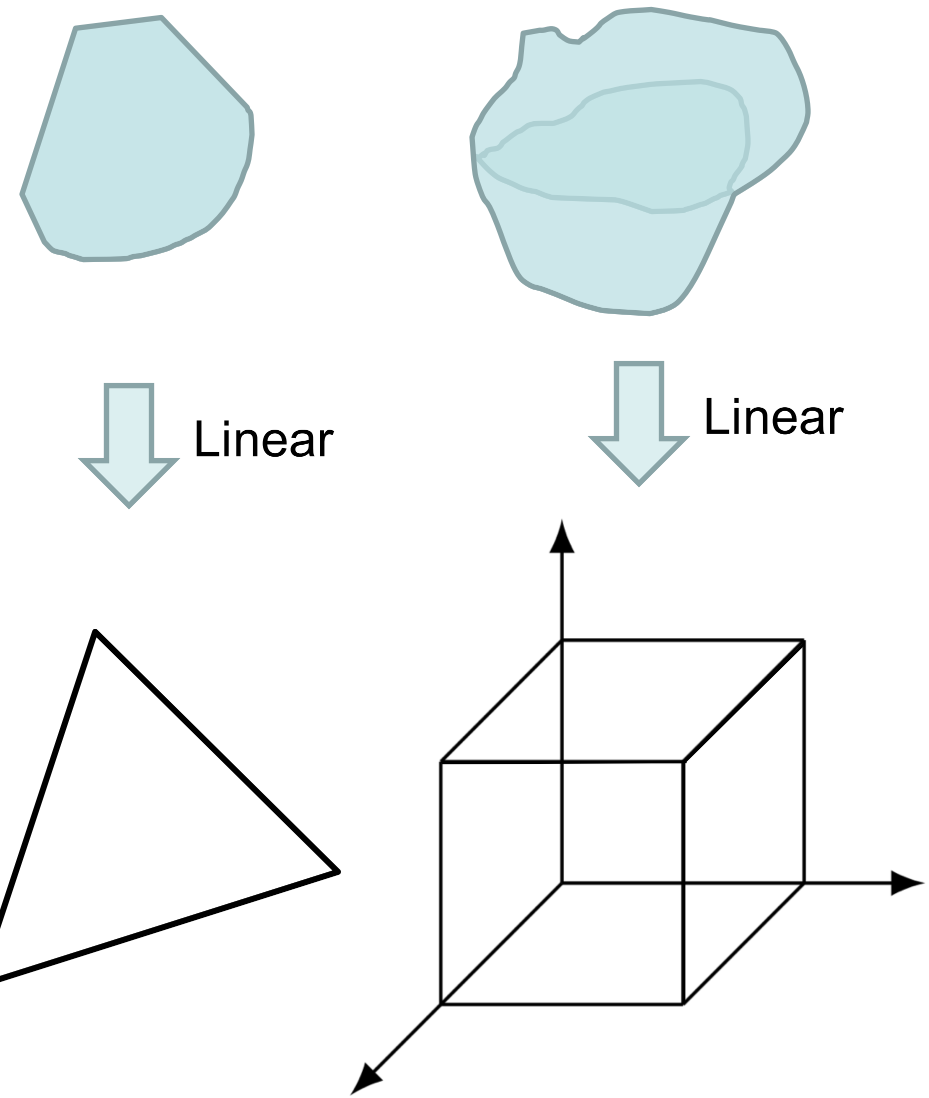
(Hilbert space dimension 3+ required)

Similarly, any theory/fragment that embeds into the classical GPT is <u>classically explainable</u>



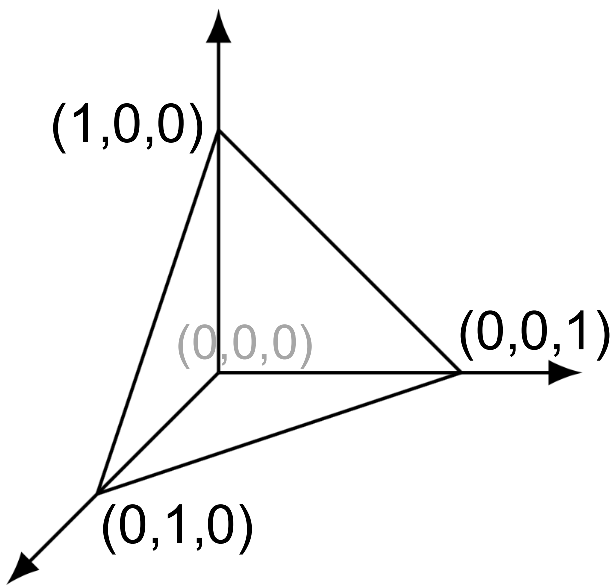
A prepare-measure GPT (fragment) is classically explainable iff there exists

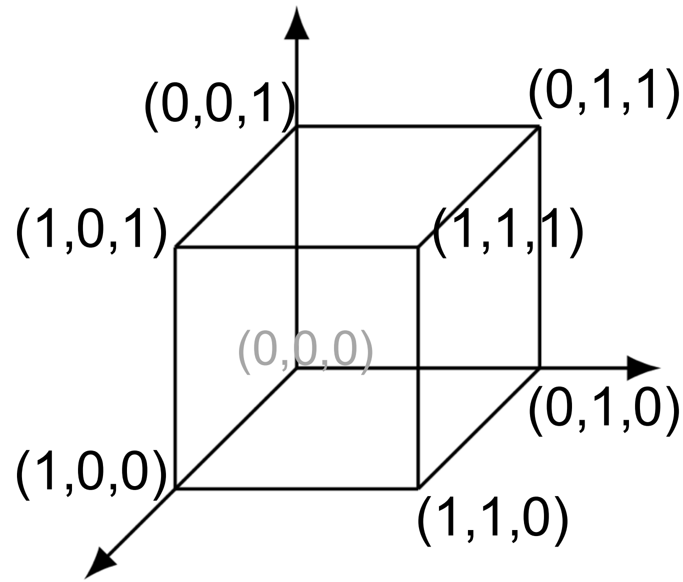
1) a linear map taking its states into a simplex, and
2) a linear map taking its effects into the dual to that simplex, such that
3) probabilities are preserved



“simplex embedding”

d=3

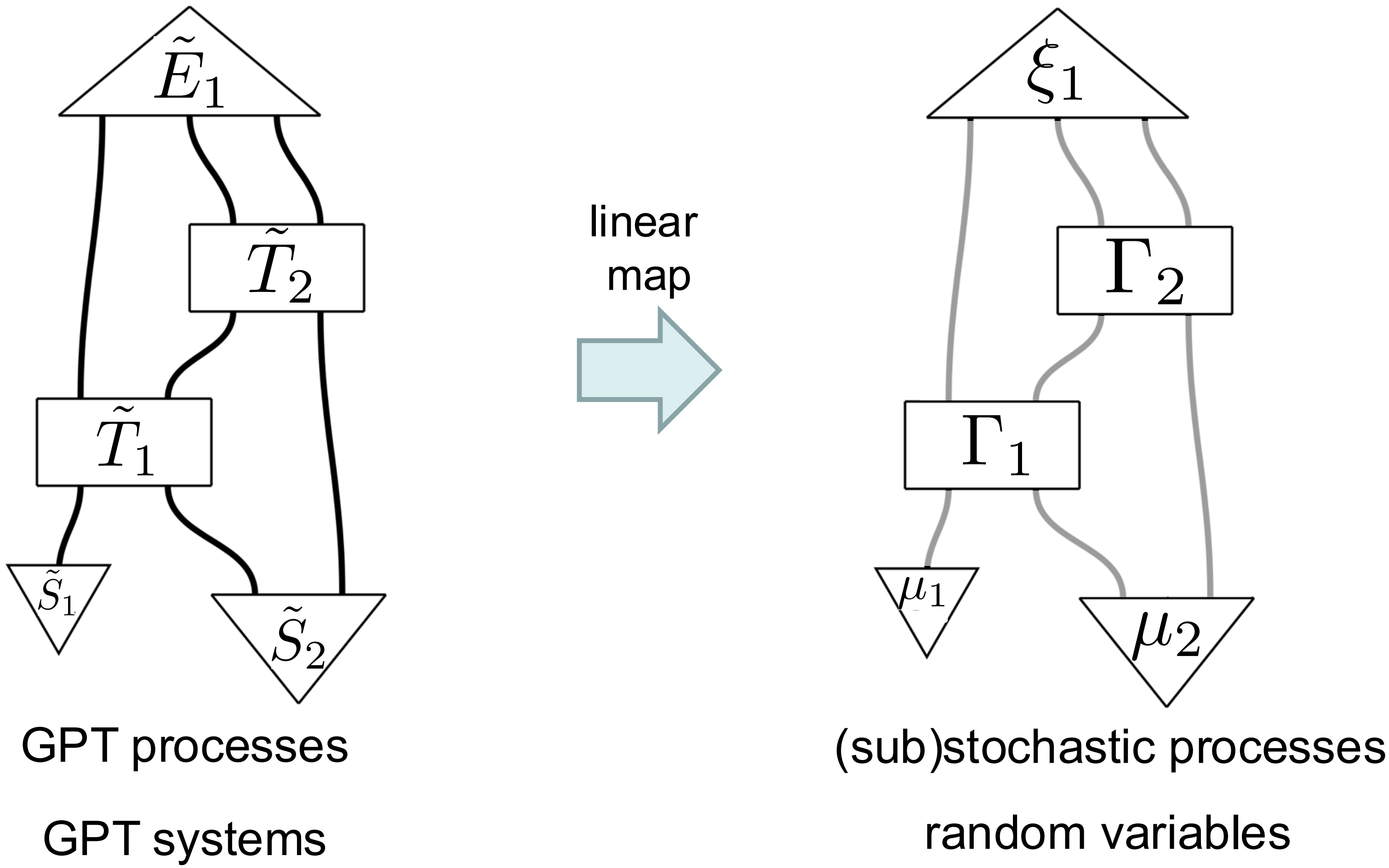




Mapping into this space gives a **positive** quasiprobability representation/ontological model for the GPT

Every quasiprobability representation is a mapping into this space (but not into the simplex or hypercube)

Beyond prepare-measure



This is *the* notion of “classical-explainability” for a GPT/fragment.

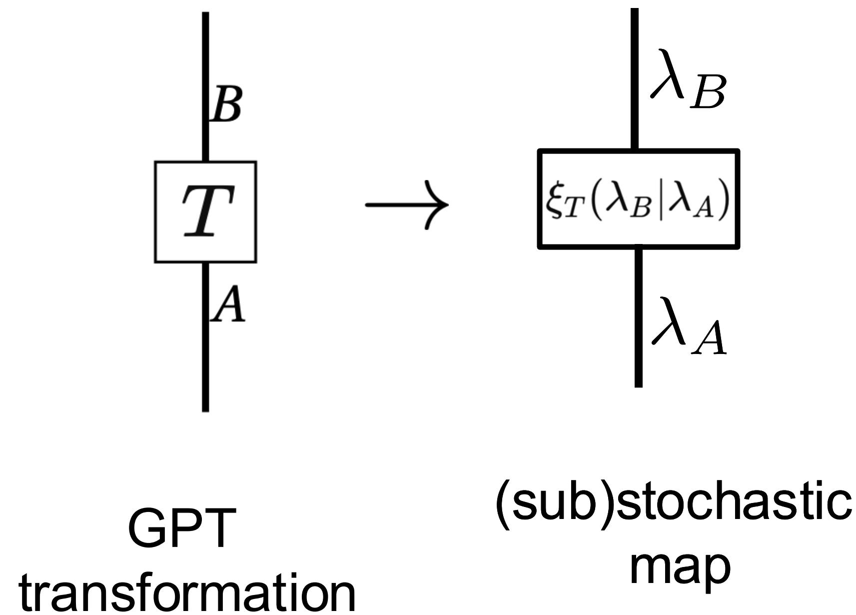
If *no* such mapping exists, the GPT/frag is *not* classically explainable.

More formally:

ξ : GPT $\to$ Classical GPT

which:

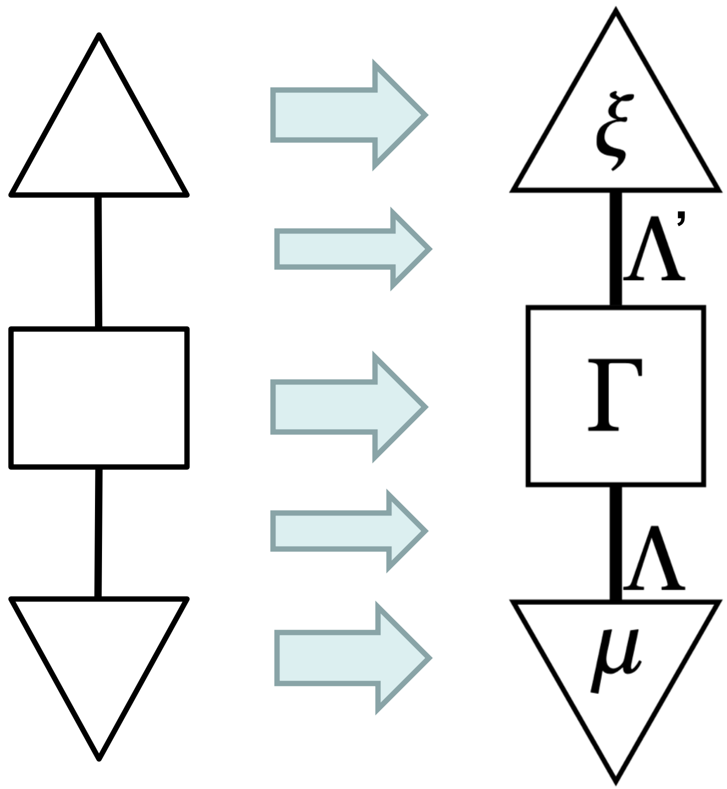
1) preserves the predictions
2) is linear
3) is diagram-preserving



= ontological model of a GPT = positive quasiprob repn

Diagram-preservation

preservation of
compositional structure



Linearity

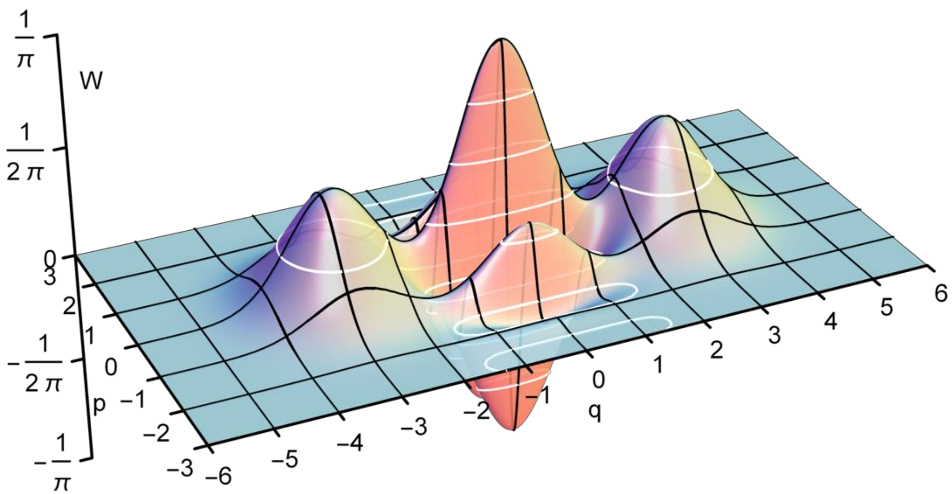
preservation of
convex geometry

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

$$\frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda) = \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$$

ex: Wigner function: linear map from quantum states to real-valued vectors

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \langle x-y|\hat{\rho}|x+y\rangle e^{2ipy/\hbar}\,dy$$



We simply generalize this to arbitrary linear functions over arbitrary classical variables

and to channels/mmts/circuits instead of just states

Then we ask whether **any** such mapping exists that is everywhere positive.

Hopefully by this stage I’ve convinced you that this is a quite fundamental notion of *classical-explainability.*

This is precisely equivalent to generalized/Spekkens' noncontextuality!

rigorous proof of nonclassicality

=

impossibility of *any* positive quasiprob repn

=

Spekkens contextuality

PRX Quantum **2**, 010331

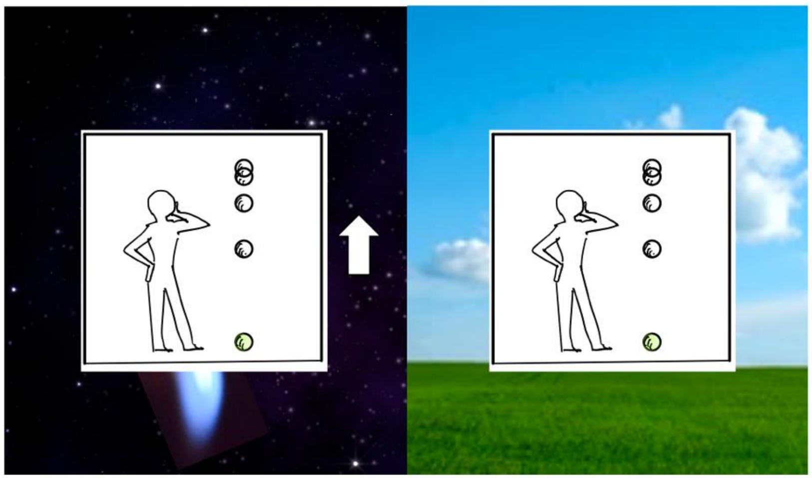
Leibniz's principle of the identity of indiscernibles—

if a difference in set-up is not distinguished in the observable phenomena then it should not be distinguished in the ontological picture either

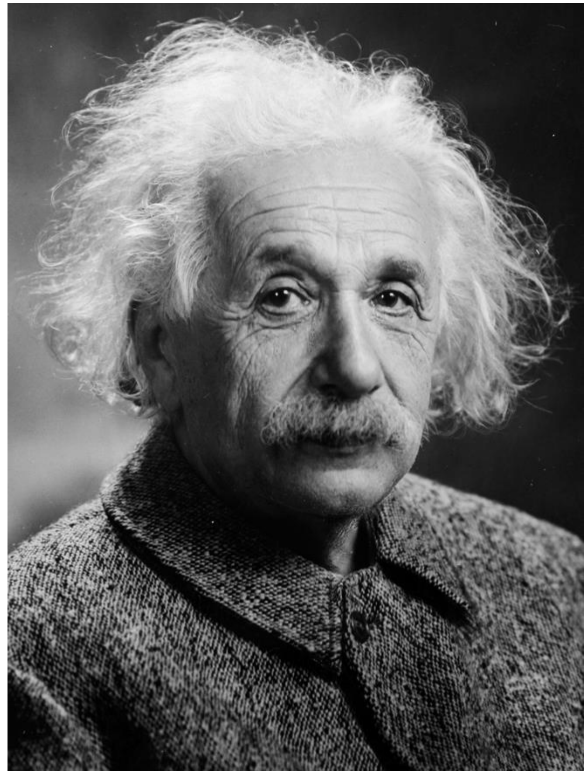
This is a methodological principle which guides us in constructing good physical theories

Leibniz’s principle in action

Einstein’s arguments against the ether



Einstein’s strong equivalence principle



indistinguishability (even in principle!)

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

$$\Big\Downarrow \text{ Linearity}$$

$$\frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda) = \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$$

sameness in the physical model

But look, if Spekkens's/Einstein's/ Leibniz's metaphysical convictions don't convince you, then refer instead to the arguments I gave here!

Things I’d love to tell you about over lunch/coffee/email:
-tools for studying noncontextuality
-conceptual insights from studying noncontextuality
-examples of contextuality in different physical contexts
-uses of contextuality as a resource for QIP

Key references:

General overview:
https://www.youtube.com/watch?v=M3qn3EHWdOg

Relating quasiprobabiltity representations,
noncontextuality, and GPTs:
PRX Quantum **2**, 010331

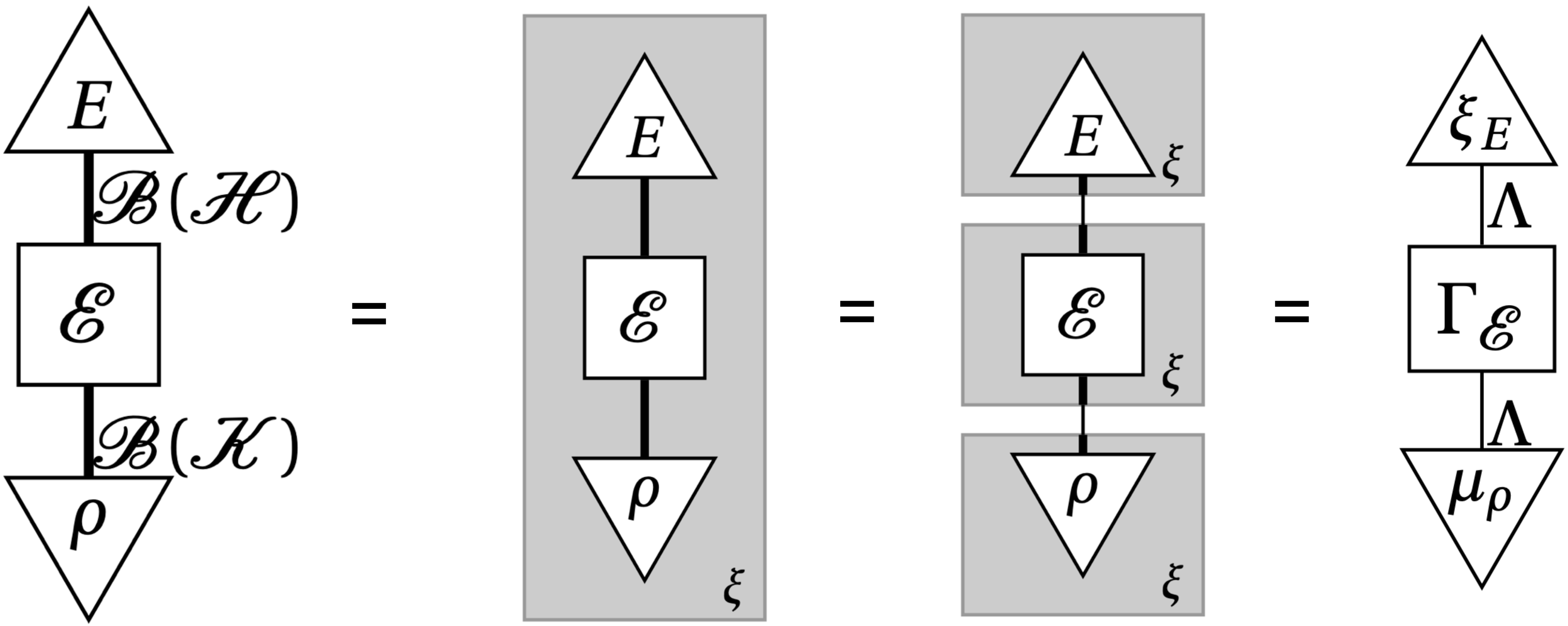
Same, but with diagrammatic tools:
Quantum 8, 1283 (2024)

Thanks for
your attention!

Diagrammatic tools for studying quasiprobability representations

Quantum 8, 1283 (2024)

A quasiprobability representation is just a (linear) diagram-preserving map from Quantum Theory to RealMat



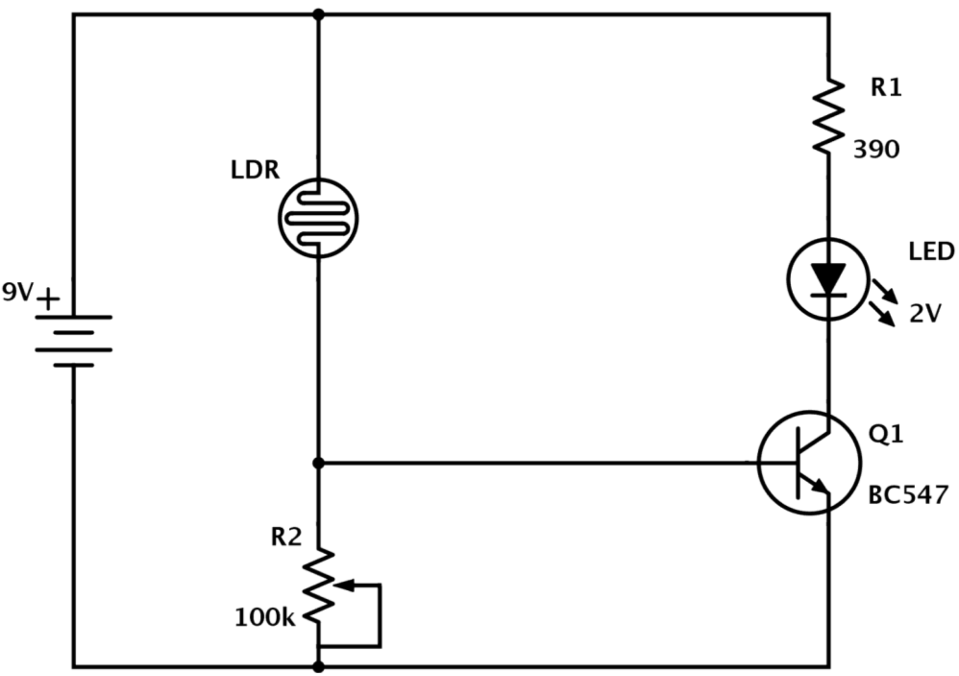
All the valuable uses of quasiprobability representations for studying properties of states, usefulness of states as resources, etc, can and should be extended to the study of measurements, channels, and arbitrary circuits/circuit fragments!

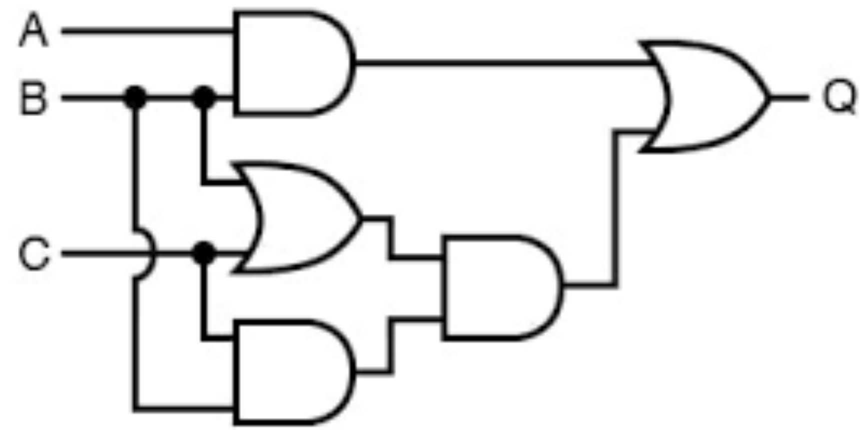
e.g., can extend Kirkwood Dirac distributions in this way

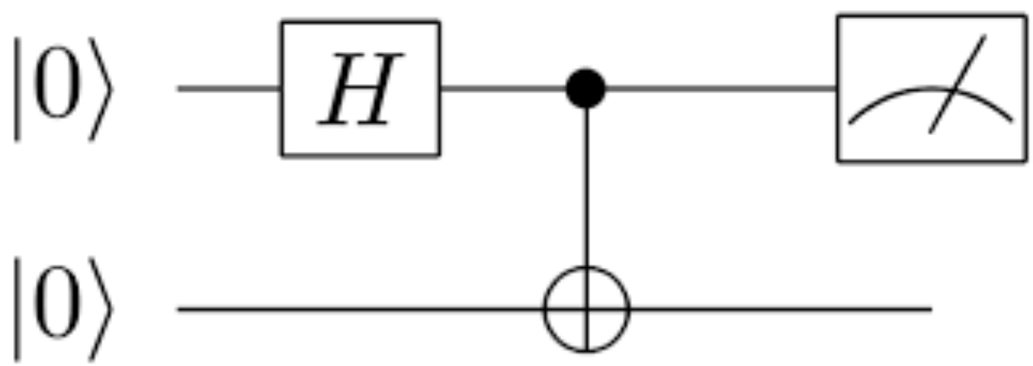
Phys. Rev. A **110**, 052206

(See Rafa's talk later today)

Circuit diagrams are intuitive and powerful…







…and can be used to do formal calculations and proofs!

Proof that every quasiprobability representation has a very specific mathematical structure

Quantum 8, 1283
arXiv:2509.10949

Proof that contextuality is a necessary resource for quantum computing (in the state injection model)

Physical Review Letters 129 (12), 120403

With additional assumptions and in specific contexts, specific quasiprobability representations might be more or less useful.

Redeeming negativity of the Wigner function (for discrete variables):

The ***unique*** positive quasiprobability repn for odd-dimensional stabilizer subtheories is Gross's discrete Wigner function Phys. Rev. Lett. 129, 120403

So any state that is negative in this repn is nonclassical (when taken together with the stabilizer subtheory)!

Analogous result in the CV case?