A STRUCTURE THEOREM FOR COMPLEX-VALUED QUASIPROBABILITY REPRESENTATIONS OF PHYSICAL THEORIES

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University of Ulm

arXiv preprint: 2509.10949

QuiDiQua^3

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PHYSICAL THEORIES

- Classical theory
- Quantum theory

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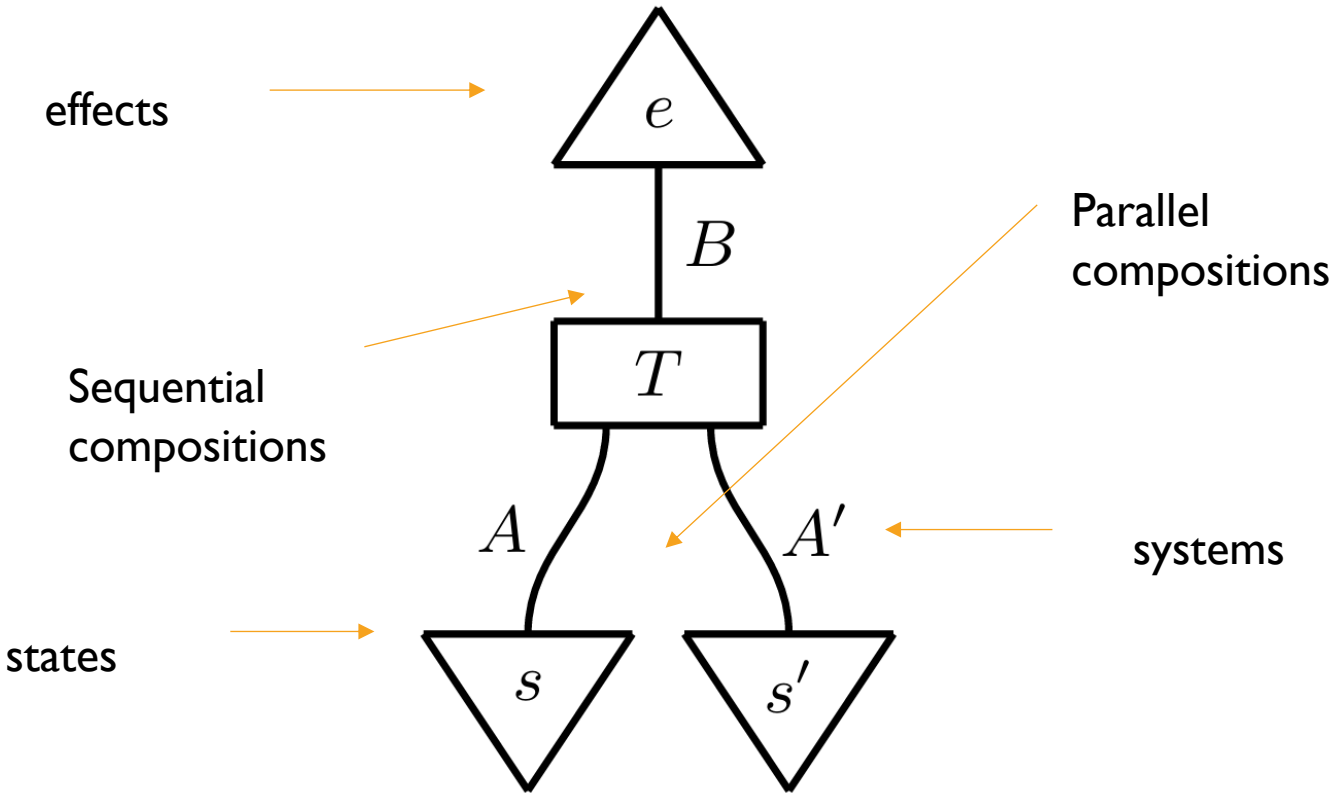
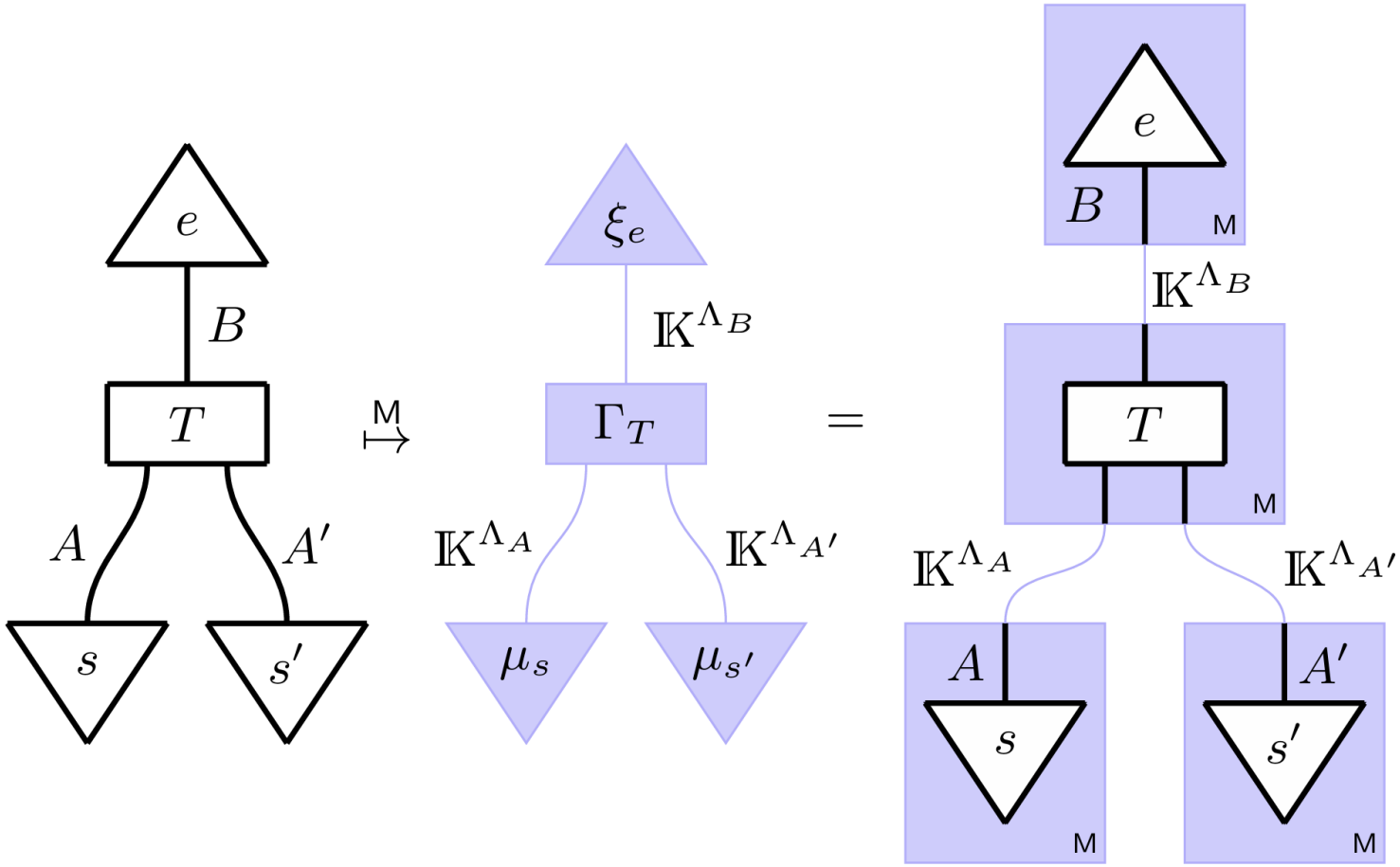
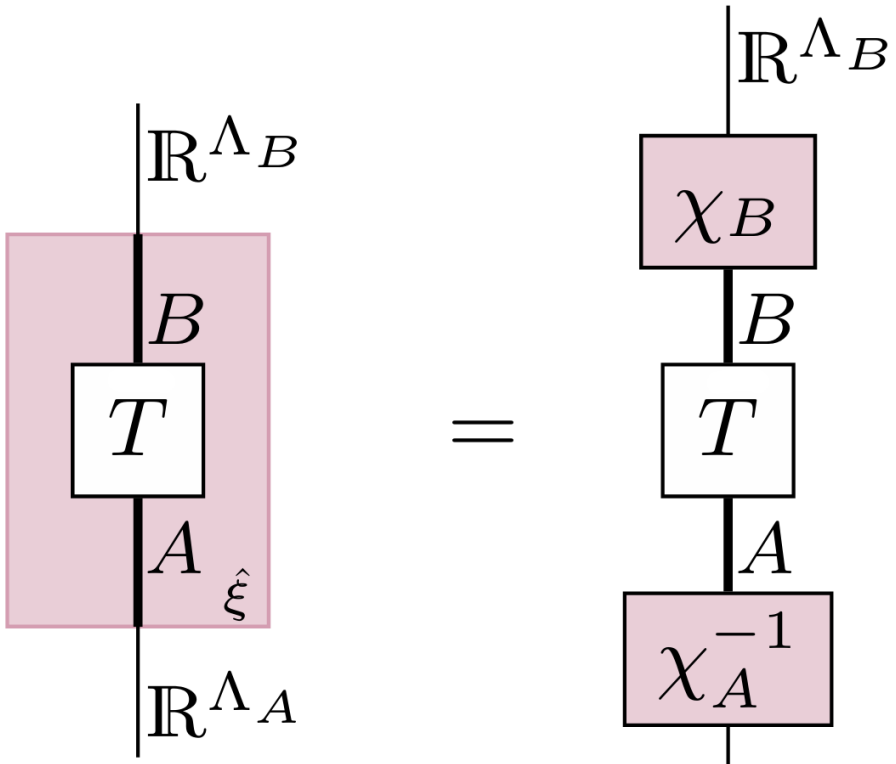


General probabilistic theories (GPTs)

QUASIPROBABILITY REPRESENTATIONS



STRUCTURE THEOREMS



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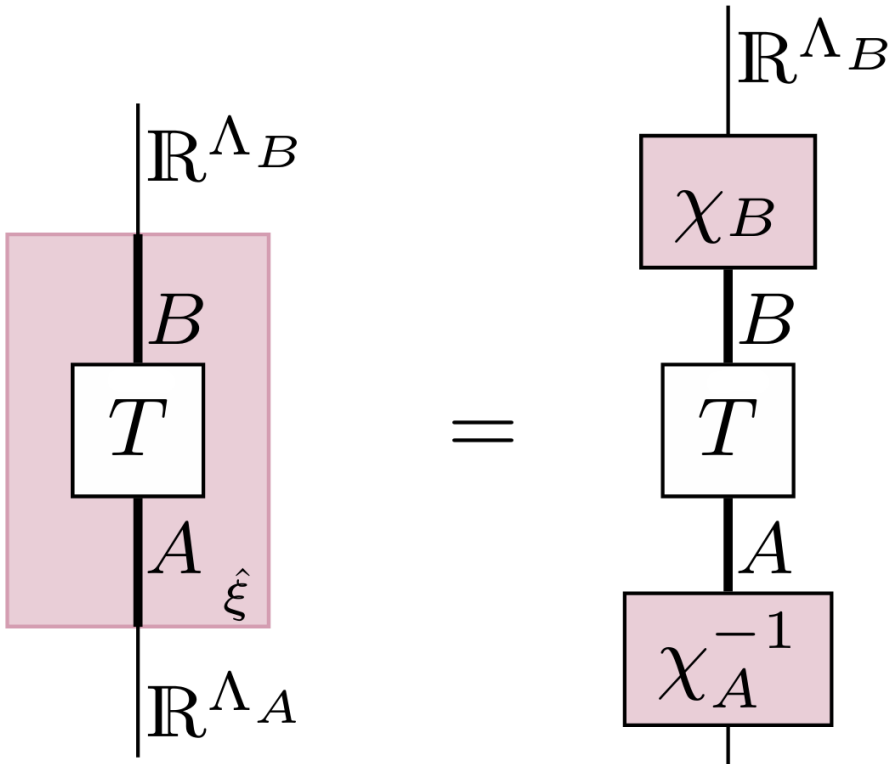
David Schmid, John H. Selby, Matthew F. Pusey,and Robert W. Spekkens, Quantum 8, 1283 (2024)

STRUCTURE THEOREMS

Goal: Extend this result



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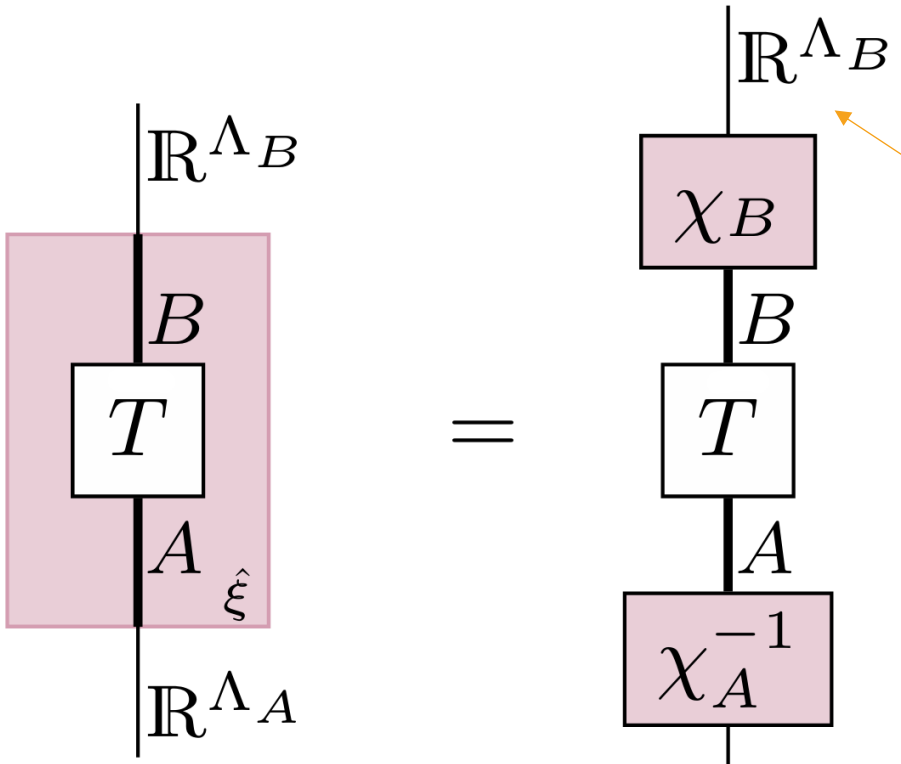

Goal: Extend this result

- Real valued

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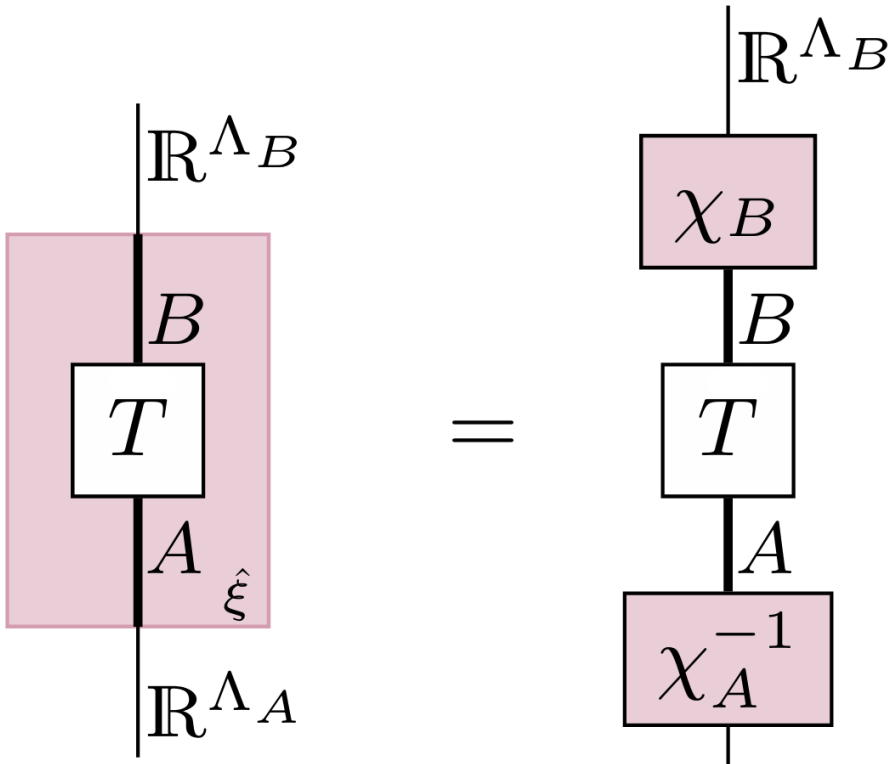

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Goal: Extend this result

- Real valued
- Finite-dimensions (target)

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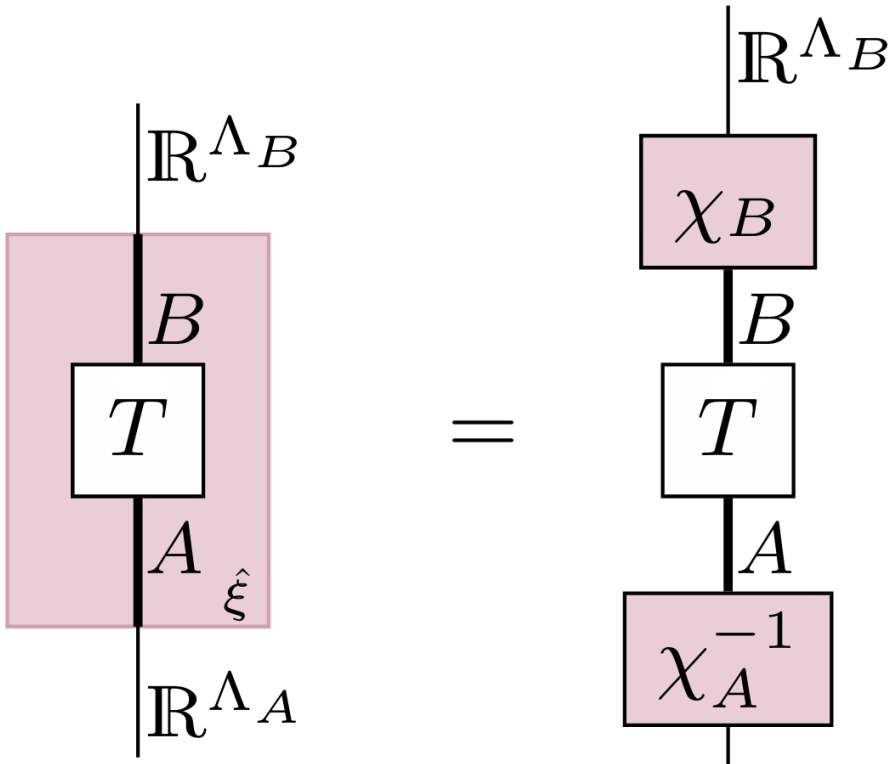
STRUCTURE THEOREMS



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Goal: Extend this result

- Real valued
- Finite-dimensions (target)
- Identity-preservation

$$\hat{\xi}(\mathrm{id}_A) = \mathrm{id}_{\mathbb{R}^{\Lambda_A}}$$

David Schmid, John H. Selby, Matthew F. Pusey,and Robert W. Spekkens, Quantum 8, 1283 (2024)

MOTIVATION

TOPICAL REVIEW • OPEN ACCESS

Properties and applications of the Kirkwood–Dirac distribution

David R M Arvidsson-Shukur*, William F Braasch Jr, Stephan De Bièvre, Justin Dressel, Andrew N Jordan, Christopher Langrenez, Matteo Lostaglio, Jeff S Lundeen and Nicole Yunger Halpern

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New Journal of Physics, Volume 26, December 2024

Citation David R M Arvidsson-Shukur *et al* 2024 *New J. Phys.* **26** 121201

DOI 10.1088/1367-2630/ada05d

$$\mu_{\mathrm{KD}}(a, b \mid \rho) = \langle a|\rho|b\rangle\langle b|a\rangle$$

David Schmid, Roberto D. Baldijao, Yile Ying, RW, John H. Selby, PRA 110, 05226 (2024)

MOTIVATION

Uniqueness of Noncontextual Models for Stabilizer Subtheories

David Schmid [1,2,3], Haoxing Du [4], John H. Selby [3], and Matthew F. Pusey [5]

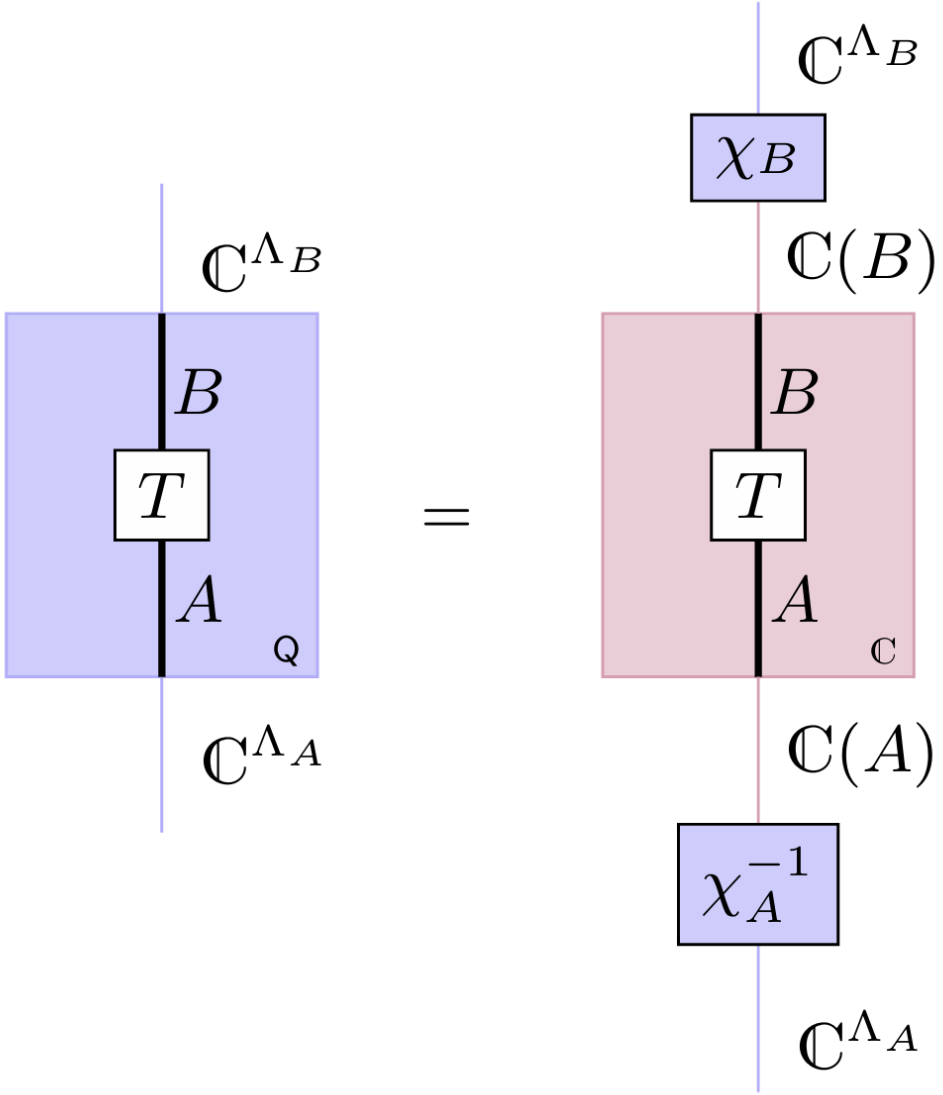
Show more

Phys. Rev. Lett. **129**, 120403 – **Published 14 September, 2022**

DOI: https://doi.org/10.1103/PhysRevLett.129.120403

Useful for connecting noncontextual models with positive quasiprobability representations for interesting applications

$$\mathbb{Q}(\mathrm{id}_A) = \mathrm{id}_{\mathbb{C}^{\Lambda_A}}$$

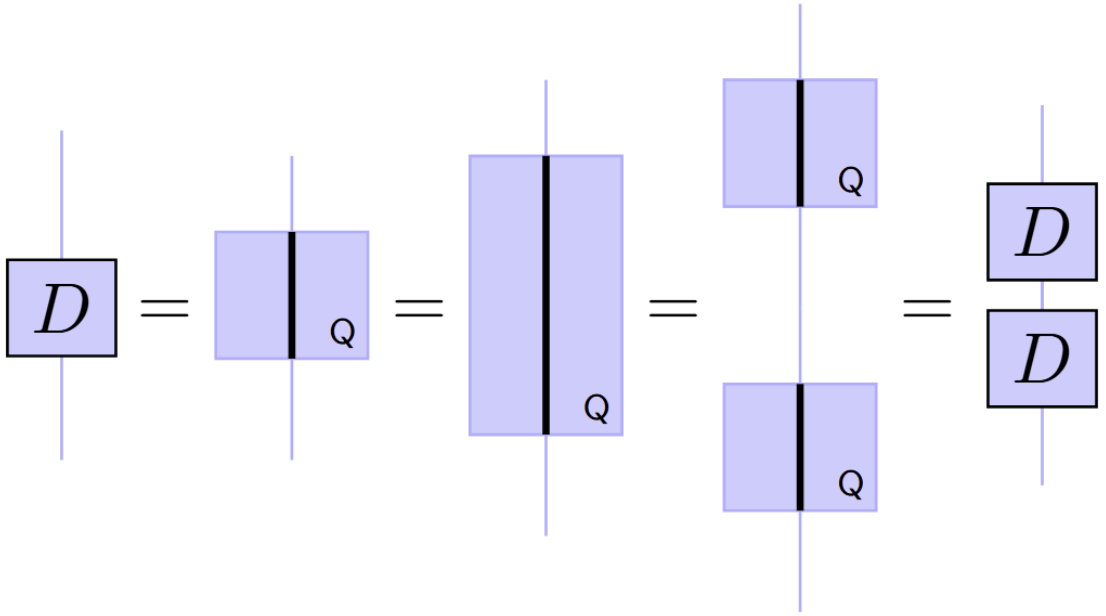


RESULTS

- ~~Real valued~~
- ~~Finite-dimensions (target)~~
- Identity-preservation

RW, Roberto D. Baldijao, Matthias Salzger, Yile Ying, David Schmid, John H. Selby, 2509.10949

$$\mathrm{Q}(\mathrm{id}_A) \neq \mathrm{id}_{\mathbb{C}^{\wedge}A}$$



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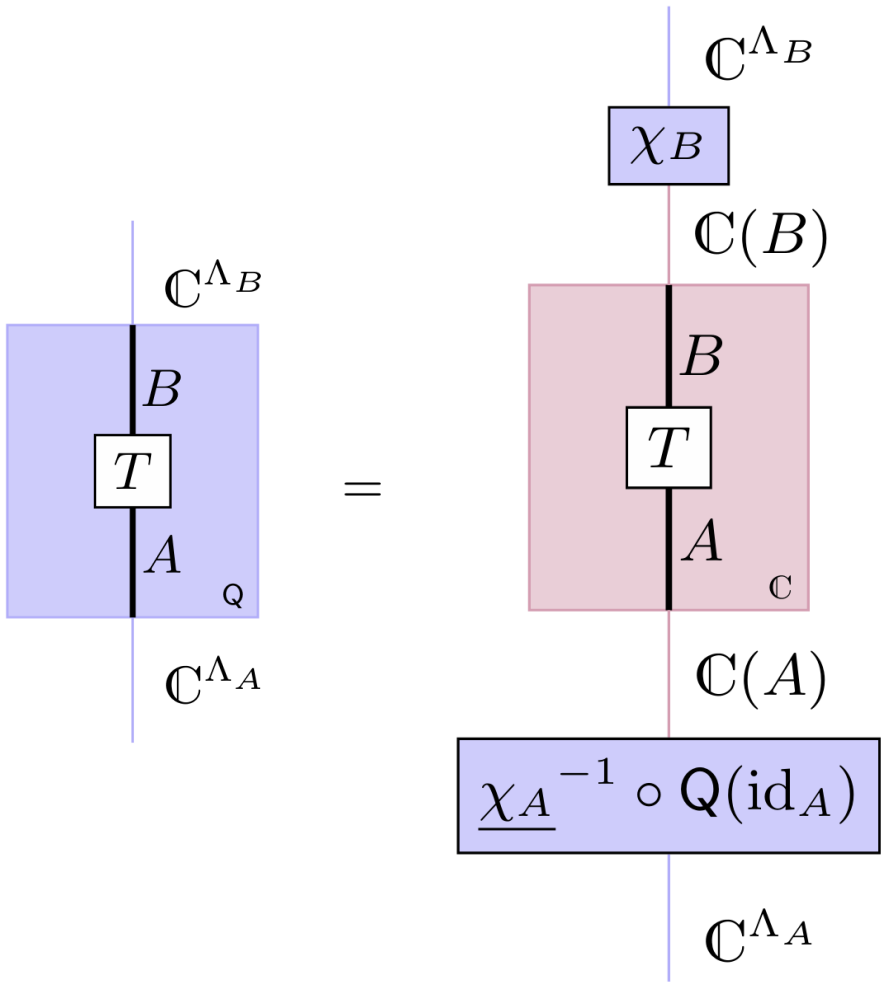

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- ~~Finite-dimensions (target)~~
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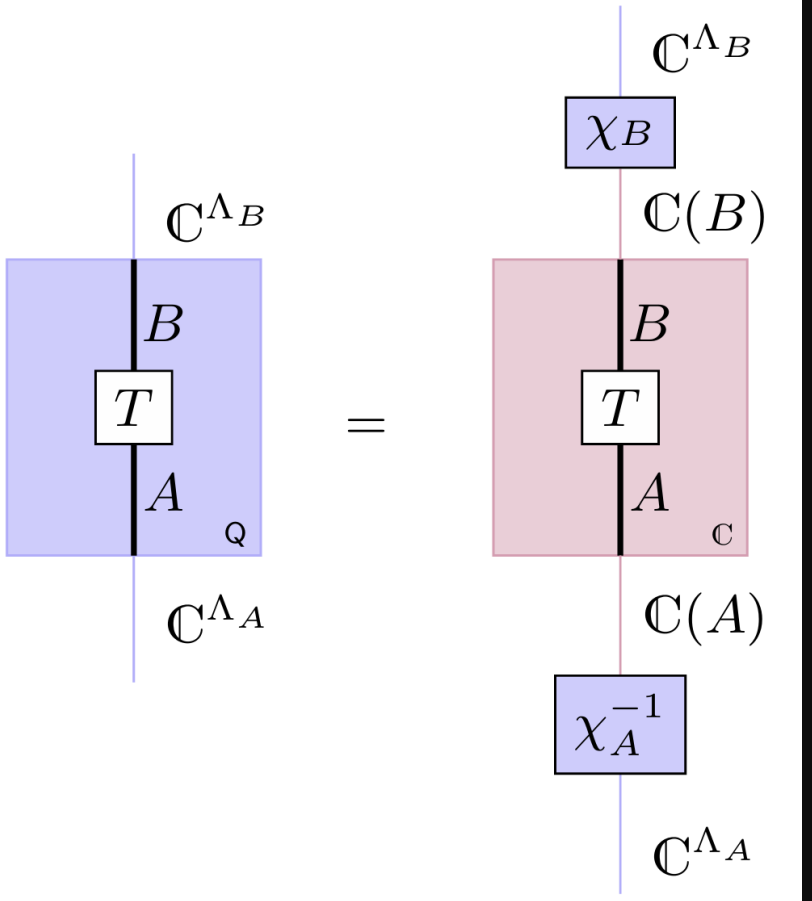
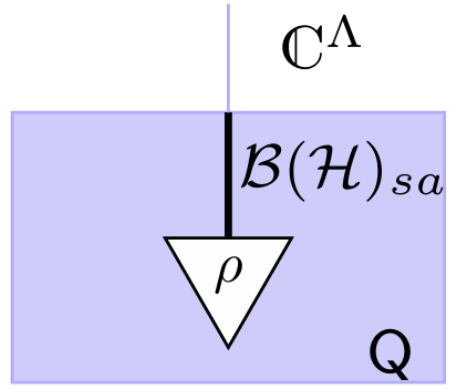
EXAMPLE



$$\mathrm{Q}(\rho) =$$



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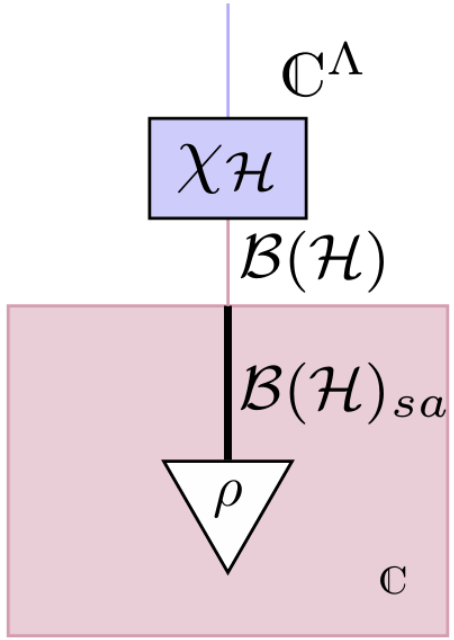

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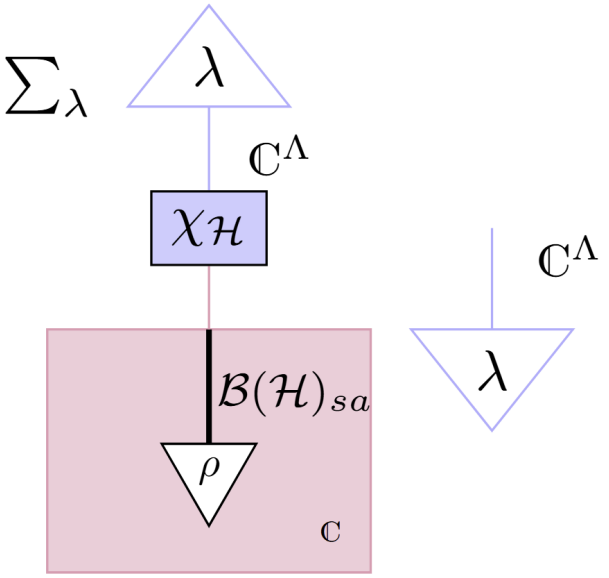
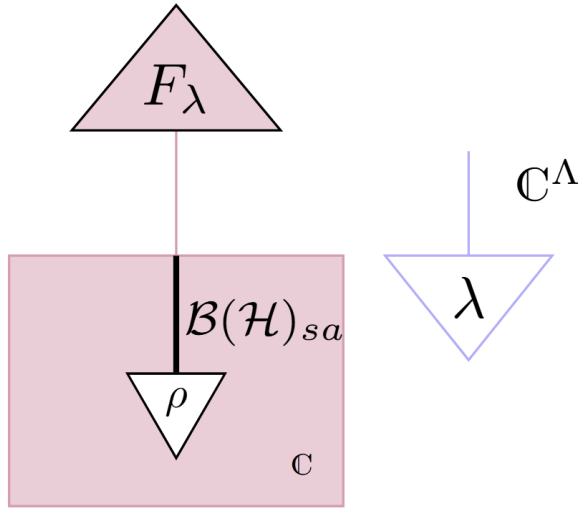
EXAMPLE

$$\mathsf{Q}(\rho) = \sum_\lambda$$



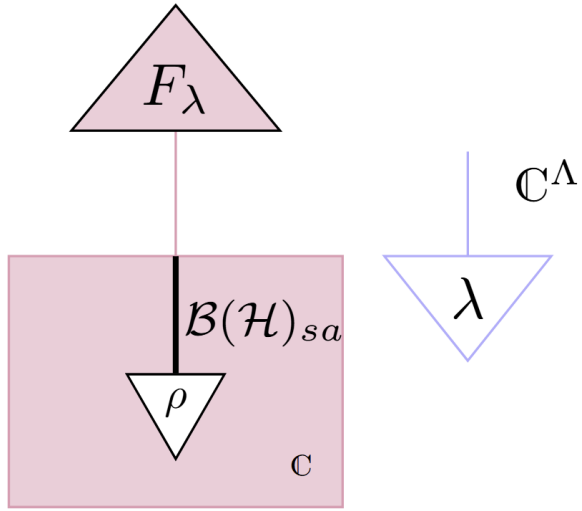
$$= \sum_\lambda$$



EXAMPLE

Frame representation theory

$$\mathsf{Q}(\rho) = \sum_\lambda$$



$$\mathrm{Tr}(F_\lambda \rho)$$

$$= \sum_{\lambda \in \Lambda} \mu(\lambda|\rho)|\lambda\rangle$$

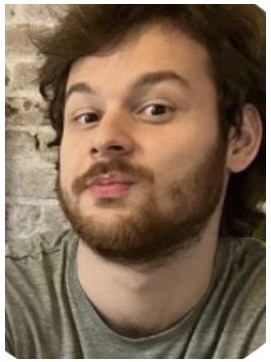
THANKS

A structure theorem for complex-valued quasiprobability representations of physical theories

Rafael Wagner,[1,*] Roberto D. Baldijão,[2,3] Matthias Salzger,[2] Yìlè Yīng,[3,4] David Schmid,[3] and John H. Selby[2]

[1] *Institute of Theoretical Physics, Ulm University, Ulm, Germany*
[2] *ICTQT – International Centre for Theory of Quantum Technologies, Gdańsk, Poland*
[3] *Perimeter Institute for Theoretical Physics, Waterloo, Canada*
[4] *Department of Physics and Astronomy, University of Waterloo, Waterloo, Canada*

(Dated: September 16, 2025)



Example

$$\mathcal{O} = \mathbf{P}(n \times n)$$

$$f(\mathbf{P}(n \times n)) = \mathbf{P}(n \times n)$$

Chi-Kwong Li, Bit-Shun Tam, Nam-Kiu Tsing, Linear Algebra and its Applications 341 5–22 (2002)

STRUCTURE THEOREMS

Example

$$\mathcal{O} = \mathbf{P}(n \times n)$$

$$f(\mathbf{P}(n \times n)) = \mathbf{P}(n \times n)$$

$$X \mapsto f(X) = A_f X B_f$$

$$X \mapsto f(X) = A_f X^{\mathrm{T}} B_f$$

$$A_f, B_f \in \mathcal{O}$$

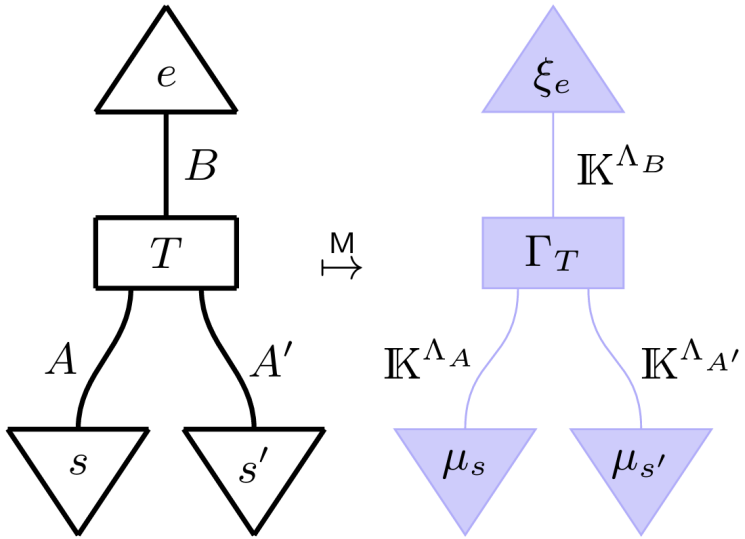
Chi-Kwong Li, Bit-Shun Tam, Nam-Kiu Tsing, Linear Algebra and its Applications 341 5–22 (2002)

STRUCTURE THEOREMS

PATH

$$\mu_{\mathrm{KD}}(a, b \mid \rho) = \langle a|\rho|b\rangle\langle b|a\rangle$$

Representing any quantum circuit



David Schmid, Roberto D. Baldijao, Yile Ying, RW, John H. Selby, PRA 110, 05226 (2024)

PATH

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Representing any quantum circuit

Any complex-valued representation

Structure theorem

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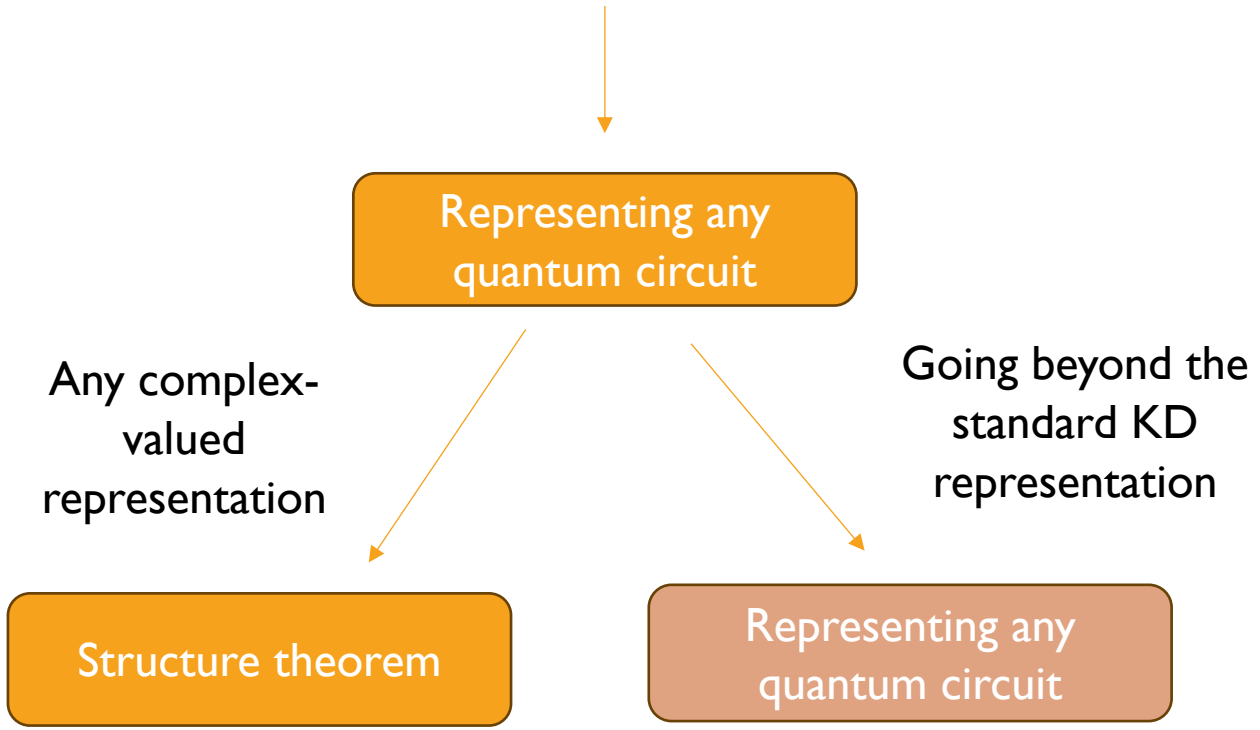

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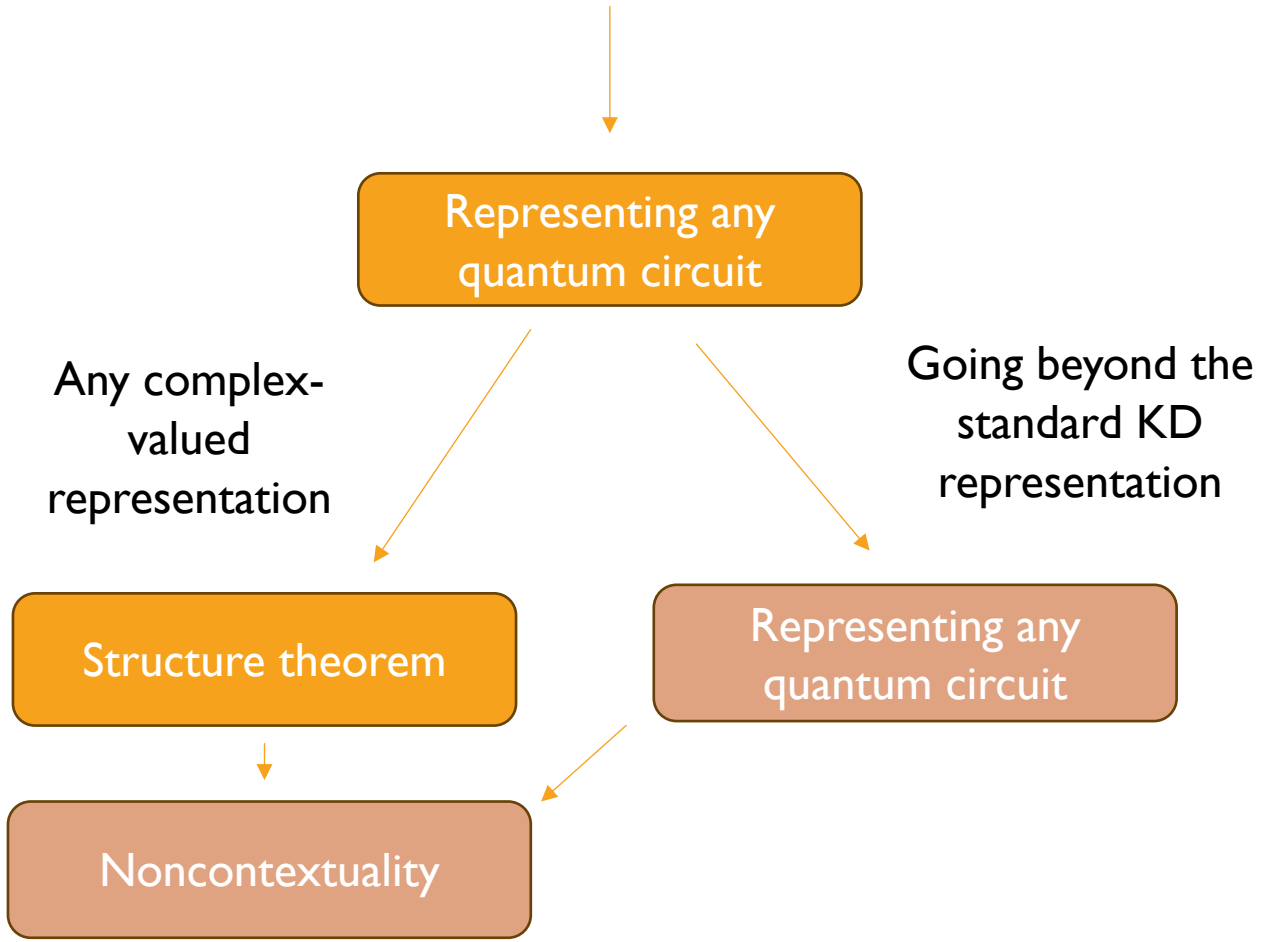

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