Squeezing, trisqueezing, and quadsqueezing in an oscillator-spin system

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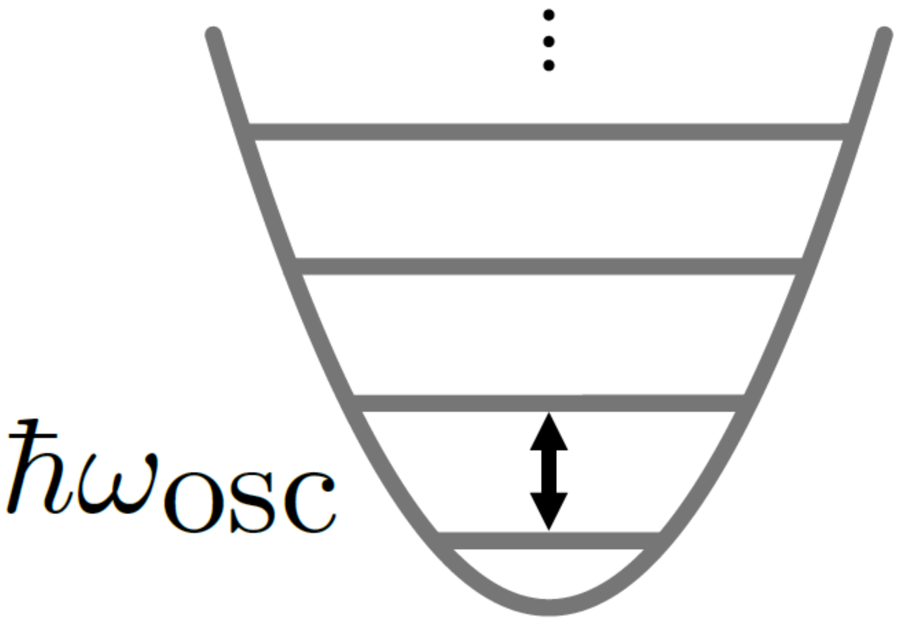
O. Băzăvan et al., arXiv:2403.05471 (2024)

Quantum harmonic oscillators



- Electromagnetic radiation

- Vibrations of particles
  - Neutral atoms
  - Nanoparticles
  - Trapped ions

- Excitations are boson

Linear interactions

$$\hat{H} \propto \hat{a} + \hat{a}^\dagger$$

- Coherent states
- Nonlinear states give rise to increasingly complex quantum behaviour

Squeezing

$$\hat{H} \propto \hat{a}^2 + \hat{a}^{\dagger 2}$$

- Minimise uncertainty in one quadrature as opposed to another

- Enhanced sensitivity
  - Gravitational wave detection [1]
  - Detection of small electric fields [2]

[1] The LIGO Scientific Collaboration, Nat. Photonics 7, 613 (2013)
[2] S. C. Burd et al., Science 364, 1163 (2019)

Generalised Squeezing

$$\hat{H} \propto \hat{a}^n + \hat{a}^{\dagger n}$$

- For n>2, states no longer Gaussian [1]
  - Harder to simulate classically
  - An essential ingredient for continuous variable quantum computation [2]
- Hard to generate
  - Might need custom hardware for specific interactions
  - State of the art is trisqueezing in superconducting systems [3, 4]

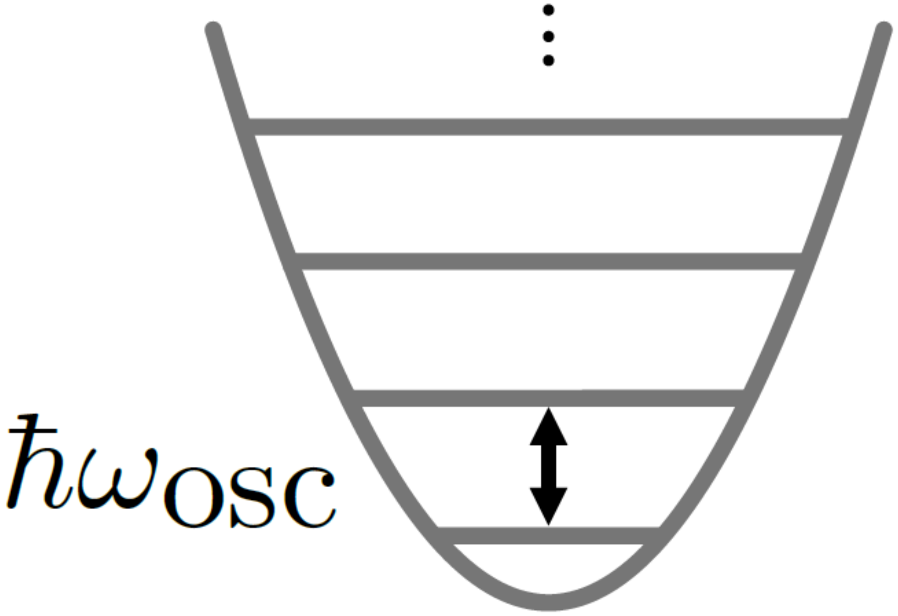
[1] S. L. Braunstein and R. I. McLachlan, Phys. Rev. A 35, 1659 (1987)
[2] S. Lloyd and S. L. Braunstein, Phys. Rev. Lett. 82, 1784 (1999)
[3] C. W. S. Chang et al., Phys. Rev. X 10, 011011 (2020)
[4] Axel M. Eriksson et al., Nat. Commun. 15, 2512 (2024)

Hard to generate

$$V(z_0) \approx V(0) + \frac{dV}{dz} z_0(\hat{a} + \hat{a}^\dagger) + \frac{1}{2}\frac{d^2V}{dz^2} z_0^2(\hat{a} + \hat{a}^\dagger)^2 + ...$$

$$\eta \propto \frac{2\pi z_0}{\lambda}$$

Hybrid oscillator-spin systems



Spin-dependent force

$$\hat{H} = \frac{\hbar\Omega_\alpha}{2}\hat{\sigma}_\alpha(\hat{a}e^{-i(\Delta\, t+\phi_\alpha)} + \hat{a}^\dagger e^{i(\Delta\, t+\phi_\alpha)})$$

- Available in a variety of platforms
  - Neutral atoms
  - Superconducting qubits
  - Colour centres
  - Trapped ions

- Typically used to generate spin-spin interactions

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Universal hybrid quantum computing in trapped ions

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Using discrete and continuous variable subsystems, hybrid approaches to quantum information could enable more quantum computational power for the same physical resources. Here, we propose a hybrid scheme that can be used to generate the necessary Gaussian and non-Gaussian operations for universal continuous variable quantum computing in trapped ions. This scheme utilizes two *linear* spin-motion interactions to generate a broad set of *nonlinear* effective spin-motion interactions including one- and two-mode squeezing, beam splitter, and trisqueezing operations in trapped ion systems. We discuss possible experimental implementations using laser-based and laser-free approaches.

Multiple spin-dependent forces

$$\hat{H} = \frac{\hbar\Omega_\alpha}{2}\hat{\sigma}_\alpha(\hat{a}e^{-i(\Delta\, t+\phi_\alpha)} + \hat{a}^\dagger e^{i(\Delta\, t+\phi_\alpha)})$$

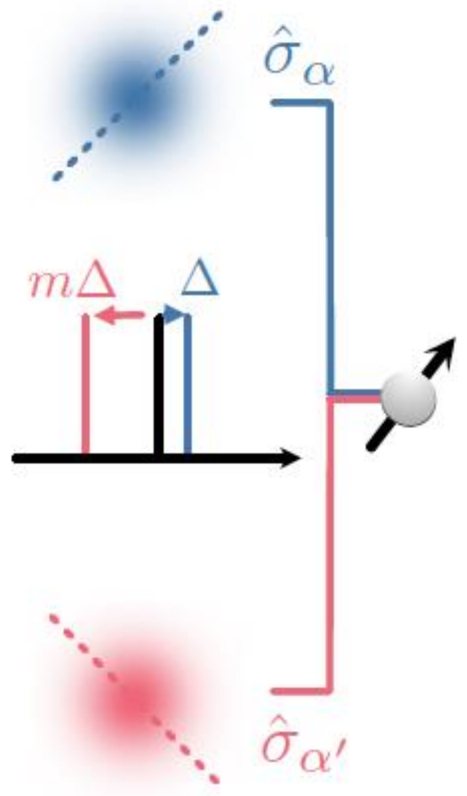
$$m = 1 - n$$

$$\hat{H}_{\mathrm{GS}} = \frac{\hbar\Omega_n}{2}\hat{\sigma}_\beta(\hat{a}^n e^{-i\theta} + \hat{a}^{\dagger n} e^{i\theta})$$

$$\Omega_n \propto \Omega_{\alpha'}\Omega_\alpha^{n-1}/\Delta^{n-1} \propto \eta$$

Generation of nonlinear interactions

$$\hat{H} \propto \hat{\sigma}_{\alpha,\alpha'}(\hat{a} + \hat{a}^\dagger)$$



Experiment

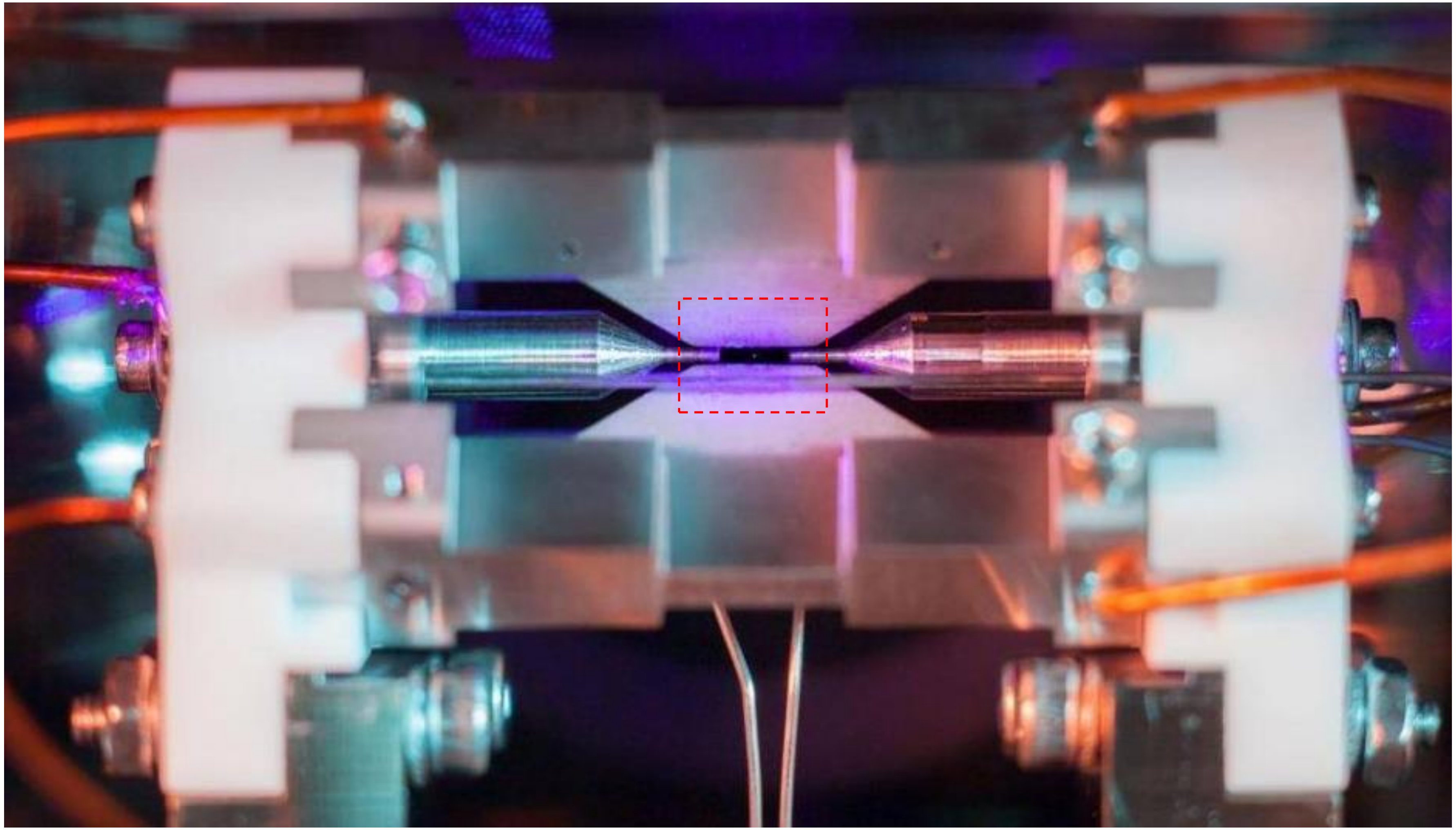
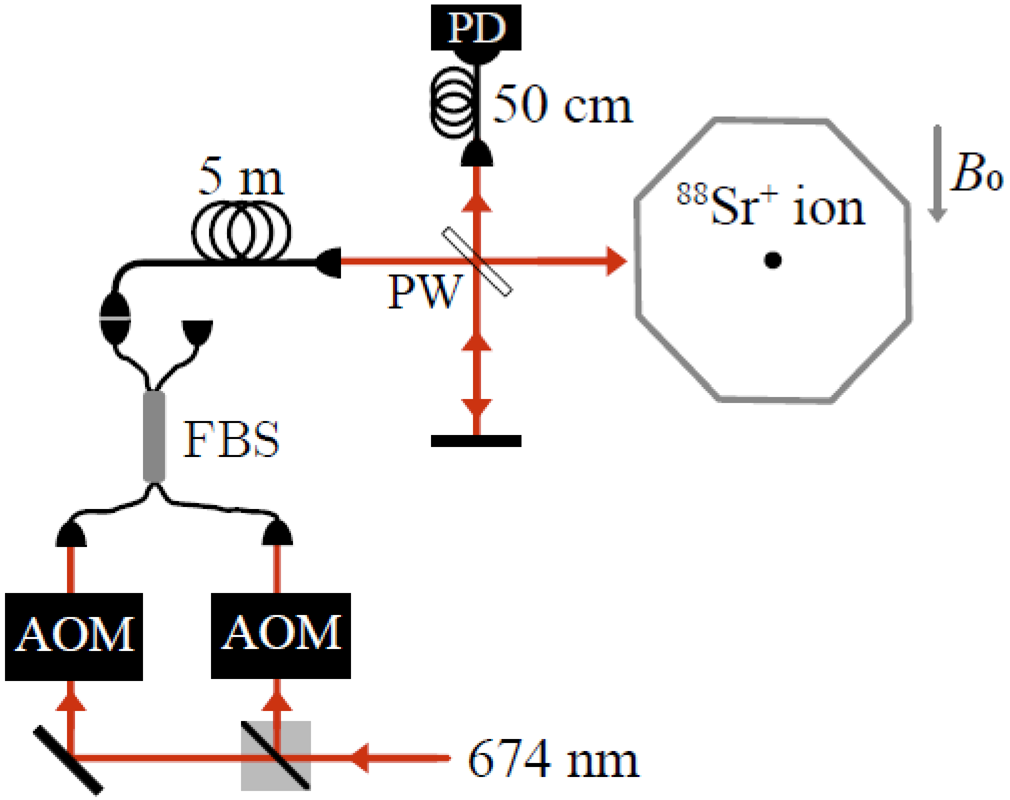


Photo credit: David Nadlinger

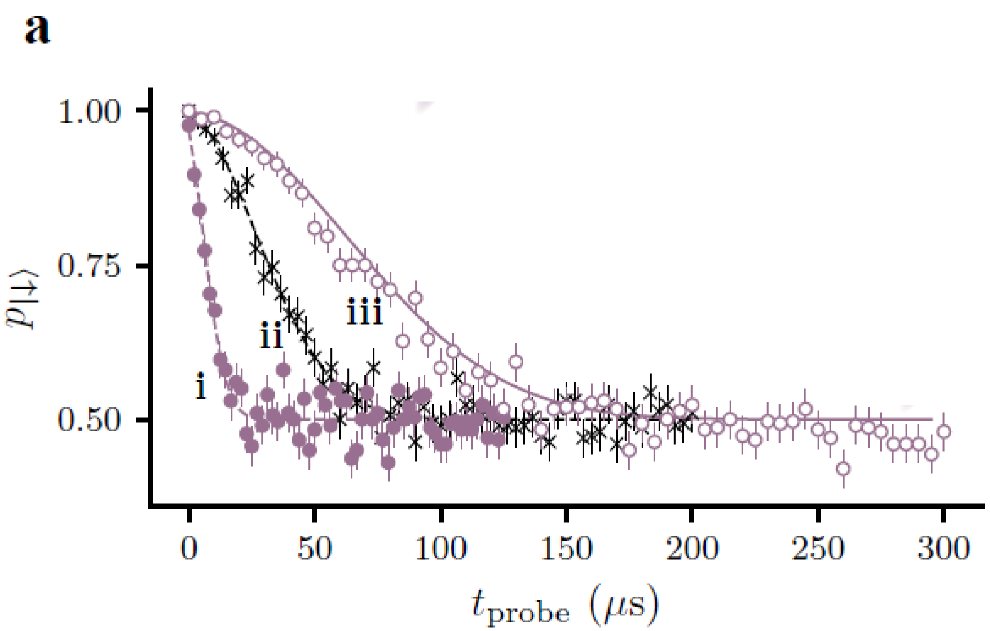
Experiment



$$\omega_{\mathrm{osc}}/2\pi \approx 1.2\ \mathrm{MHz}$$
$$\hat{\sigma}_\phi, \Omega_{\alpha,\alpha'}/2\pi \approx 4.9 - 6.9\ \mathrm{kHz}$$
$$\hat{\sigma}_z, \Omega_{\alpha'}/2\pi \approx 1.3\ \mathrm{kHz}$$

Spin-dependent squeezing

$$\hat{H}_2 = \frac{\hbar\Omega_2}{2}\hat{\sigma}_z(\hat{a}^2 e^{-i\theta} + \hat{a}^{\dagger 2} e^{i\theta})$$

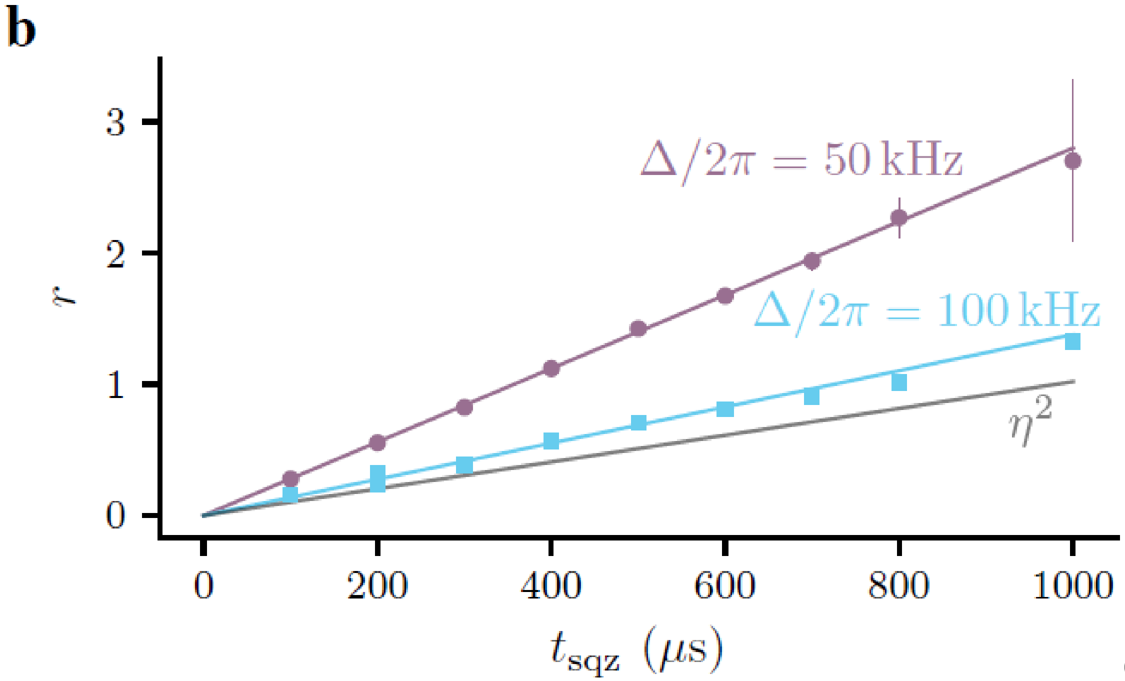


O. Băzăvan et al., arXiv:2403.05471 (2024)

Spin-dependent squeezing - amplitude

$$\hat{H}_2 = \frac{\hbar\Omega_2}{2}\hat{\sigma}_z(\hat{a}^2 e^{-i\theta} + \hat{a}^{\dagger 2} e^{i\theta})$$

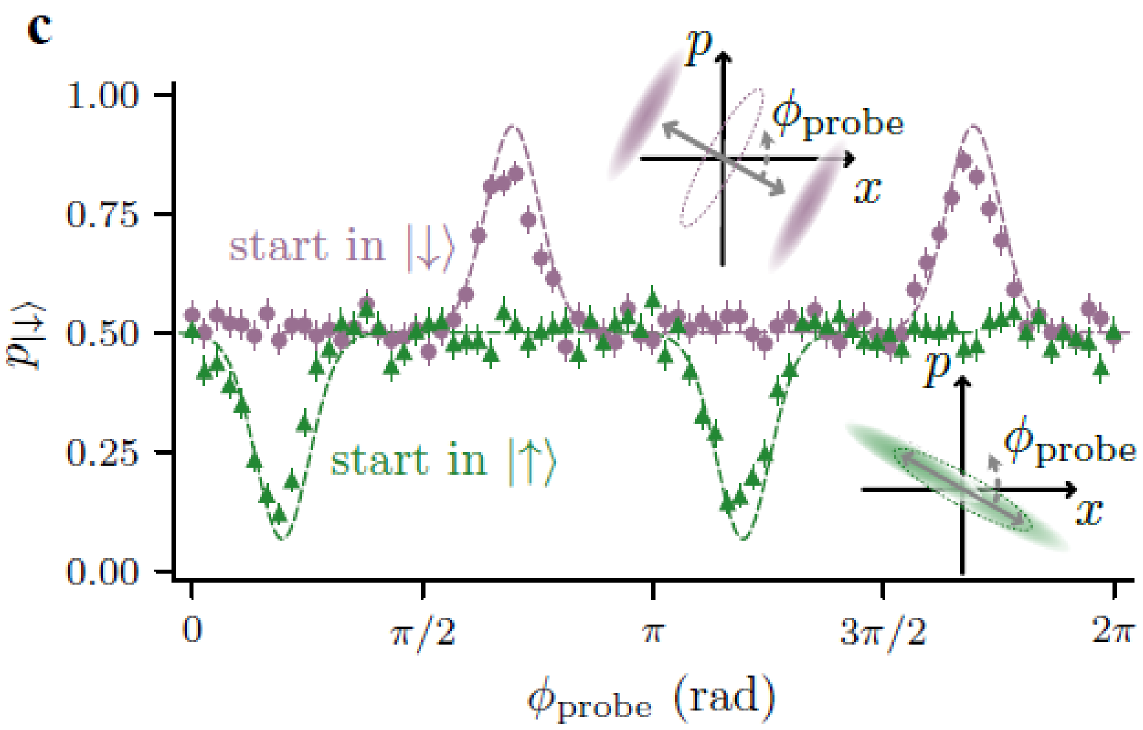
$$\Omega_2 = \frac{\Omega_{\alpha'}\Omega_\alpha}{\Delta}$$



O. Băzăvan et al., arXiv:2403.05471 (2024)

Spin-dependent squeezing – spin dependence

$$\hat{H}_2 = \frac{\hbar\Omega_2}{2}\hat{\sigma}_z(\hat{a}^2 e^{-i\theta} + \hat{a}^{\dagger 2} e^{i\theta})$$

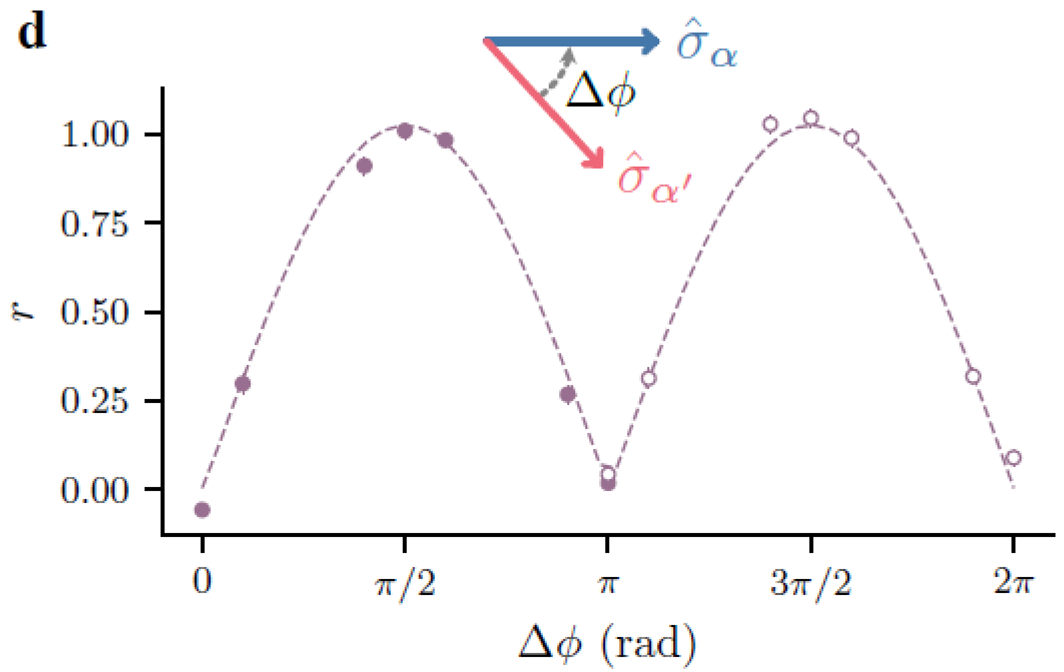


O. Băzăvan et al., arXiv:2403.05471 (2024)

Spin-dependent squeezing – non-commutativity

$$\hat{H}_2 = \frac{\hbar\Omega_2}{2}\hat{\sigma}_z(\hat{a}^2 e^{-i\theta} + \hat{a}^{\dagger 2} e^{i\theta})$$

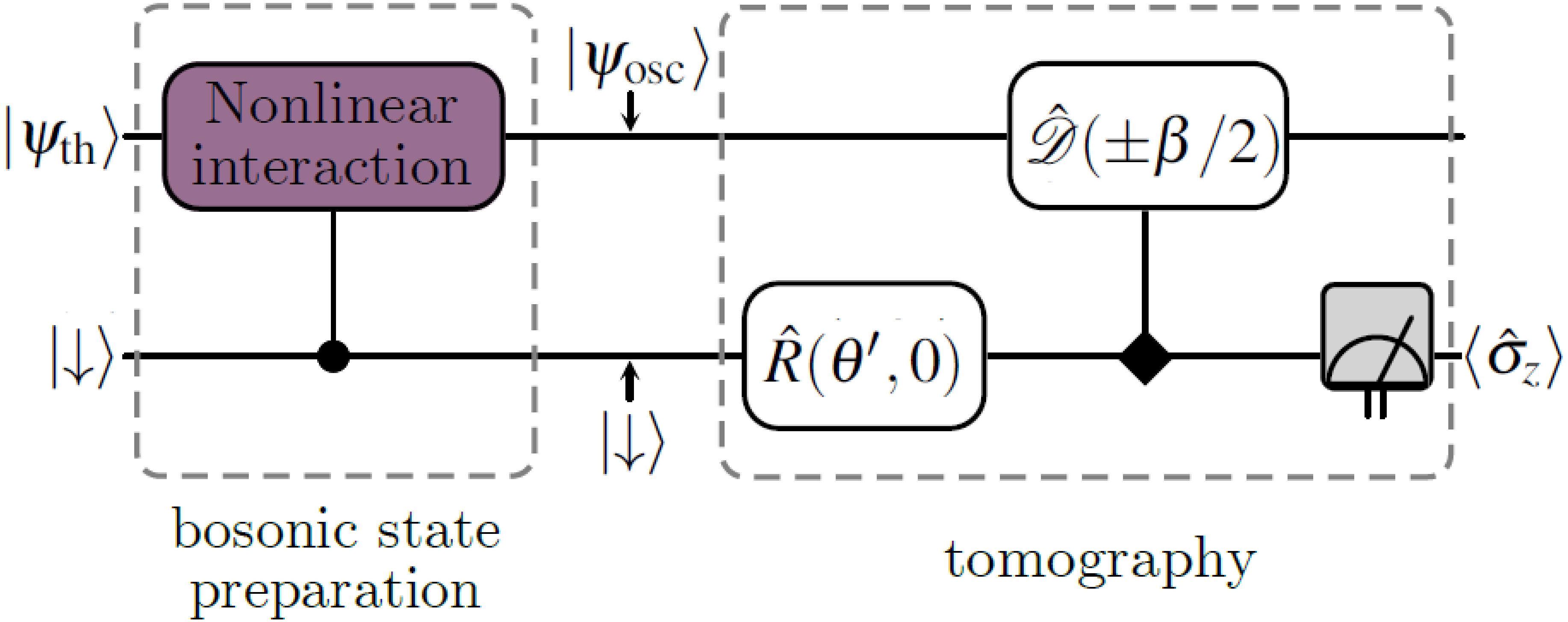
$$\hat{\sigma}_\beta \propto [\hat{\sigma}_\alpha, \hat{\sigma}_{\alpha'}]$$



O. Băzăvan et al., arXiv:2403.05471 (2024)

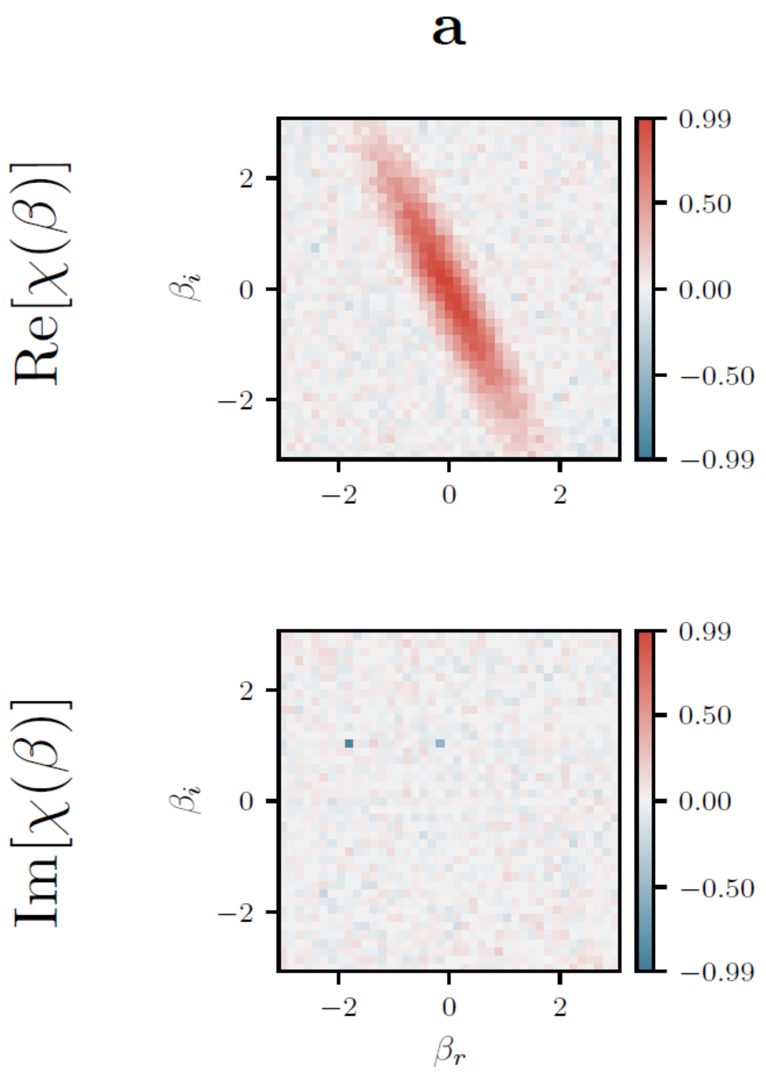
Characteristic function

$$\chi(\beta) = \langle \psi_{\mathrm{osc}} | \hat{\mathcal{D}}(\beta) | \psi_{\mathrm{osc}} \rangle$$



C. Flühmann and J. P. Home, Phys. Rev. Lett. 125, 043602 (2020)

Characteristic function

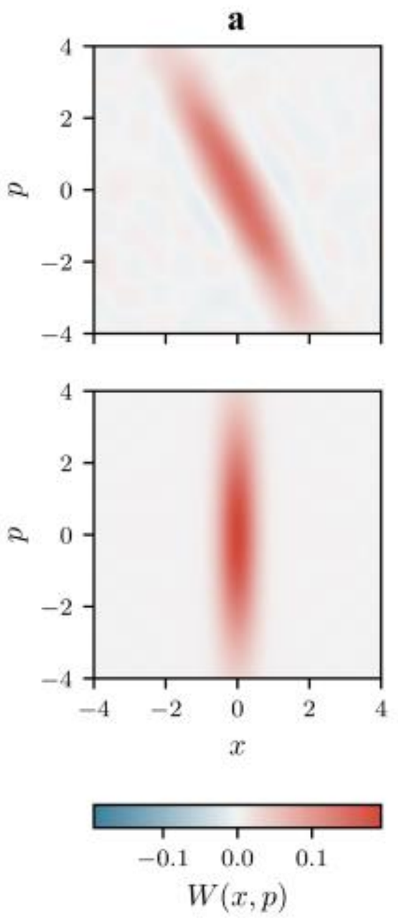


Wigner function

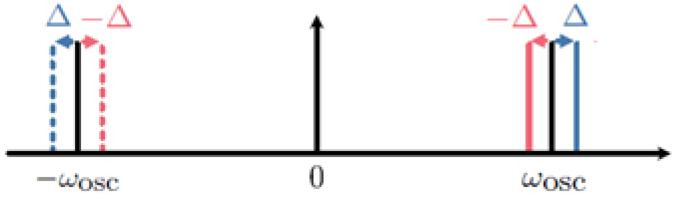
$$W(x,p) = \frac{1}{2\pi^2}\int\int \chi(\beta_r,\beta_i)e^{-2i\frac{x\beta_i - p\beta_r}{\sqrt{2}}}d\beta_r d\beta_i$$

experiment

theory



squeezing



$$\Delta/2\pi = 50\,\mathrm{kHz}$$
$$t_{sqz} = 400\,\mu\mathrm{s}$$
$$r_2 = 1.09\,(4)$$

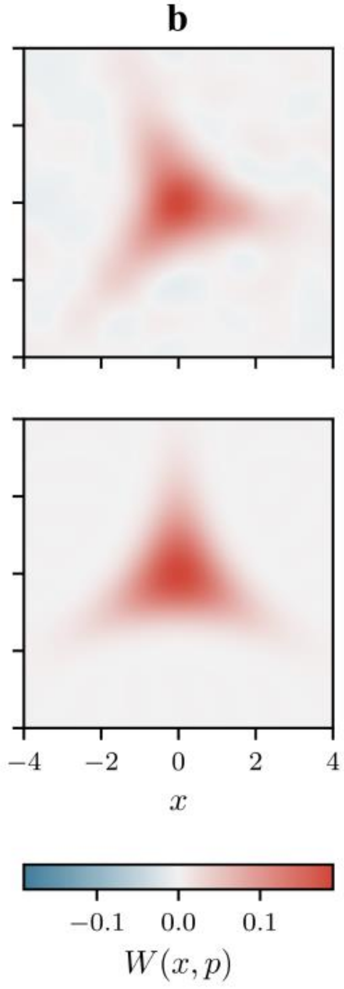
O. Băzăvan et al., arXiv:2403.05471 (2024)

Trisqueezing

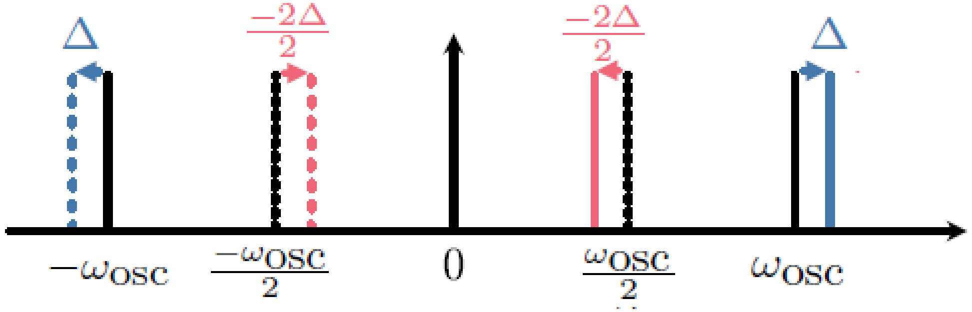
$$\hat{H}_3 = \frac{\hbar\Omega_3}{2}\hat{\sigma}_z(\hat{a}^3 e^{-i\theta} + \hat{a}^{\dagger 3} e^{i\theta})$$

experiment

theory



trisqueezing



$$\Delta/2\pi = 25\,\mathrm{kHz}$$

$$t_{\mathrm{sqz}} = 600\,\mu\mathrm{s}$$

$$r_3 = 0.19\,(1)$$

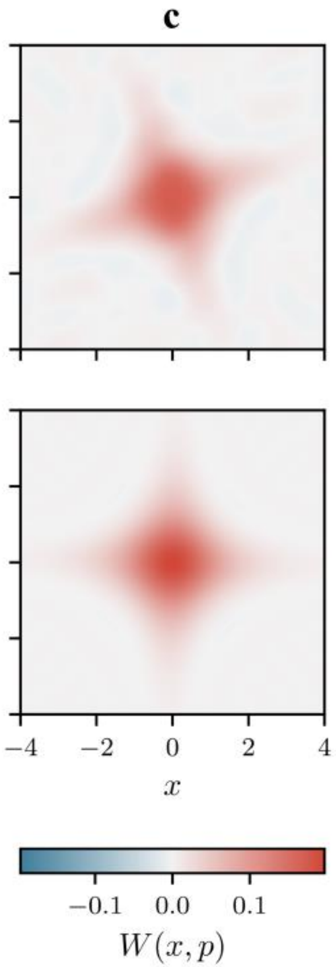
O. Băzăvan et al., arXiv:2403.05471 (2024)

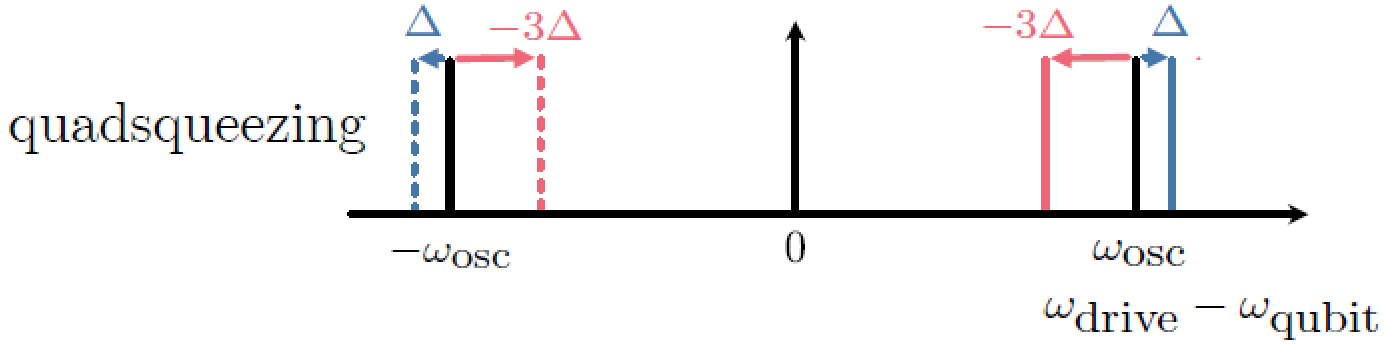
Quadsqueezing

$$\hat{H}_4 = \frac{\hbar\Omega_4}{2}\hat{\sigma}_z\left(\hat{a}^4 e^{-i\theta} + \hat{a}^{\dagger 4} e^{i\theta}\right)$$

experiment

theory





$$\Delta/2\pi = 25\,\mathrm{kHz}$$

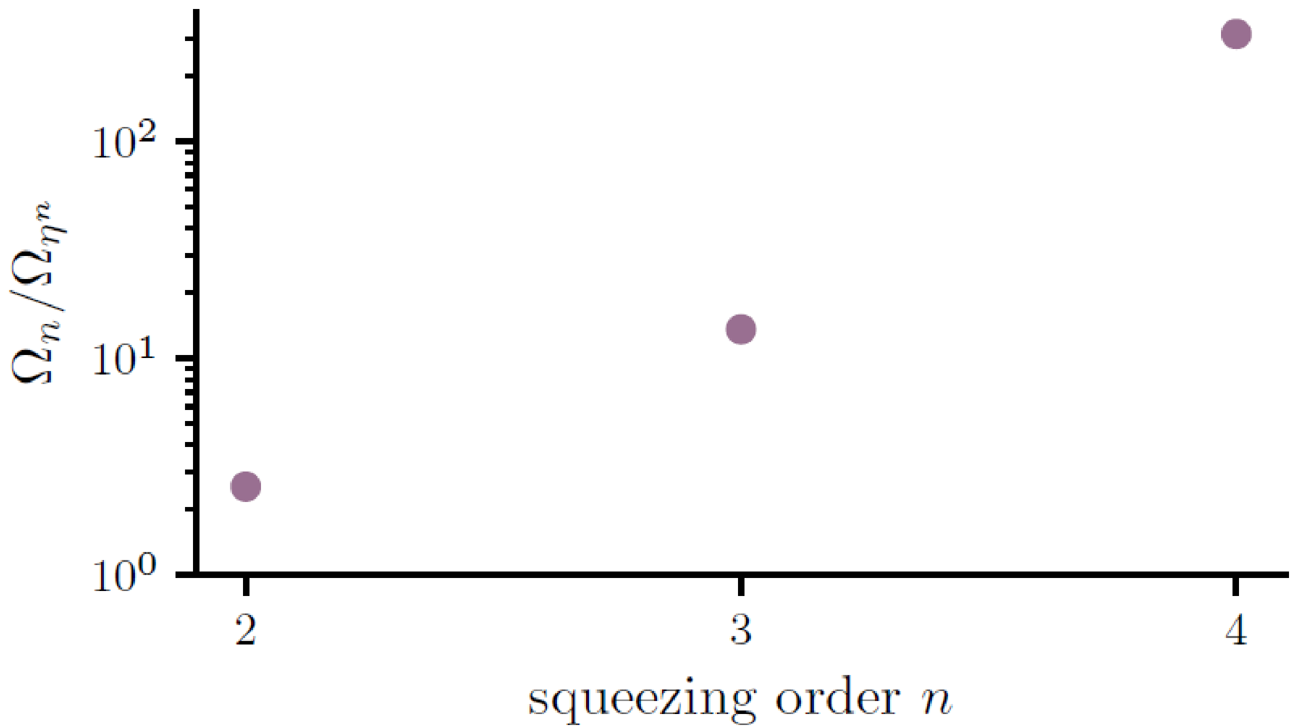
$$t_{\mathrm{sqz}} = 600\,\mu\mathrm{s}$$

$$r_4 = 0.054\,(5)$$

O. Băzăvan et al., arXiv:2403.05471 (2024)

Quadsqueezing

Interaction strength

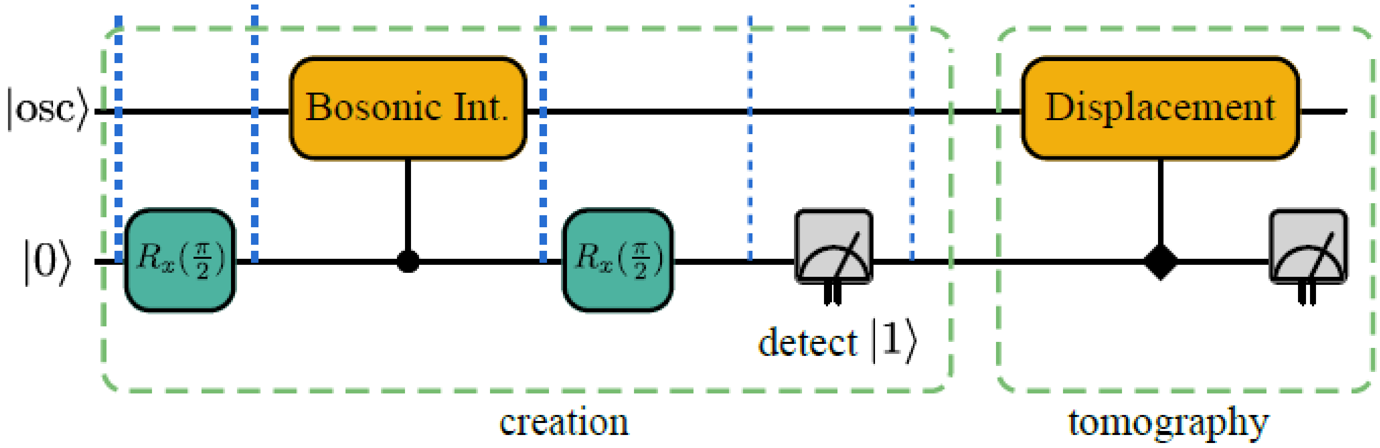


Generation of arbitrary superpositions of nonclassical oscillator states

Sebastian Saner

S. Saner et al., arXiv:2409.03482 (2024)

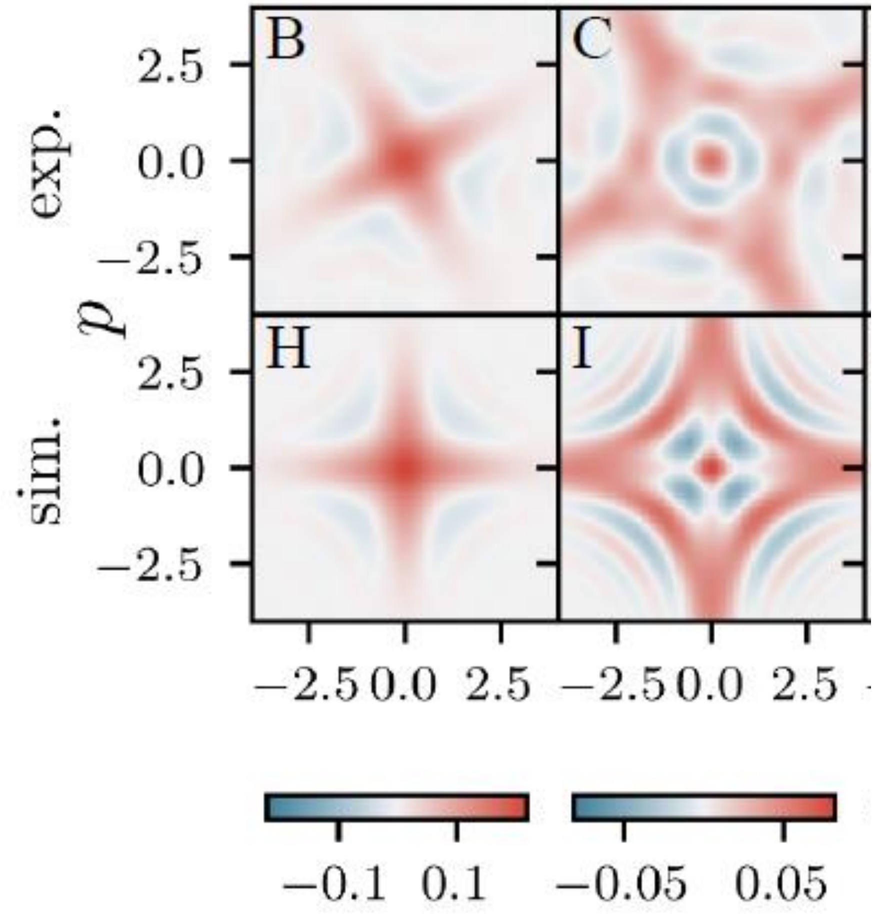
Protocol



|osc⟩
|0⟩
Bosonic Int.
$R_x(\frac{\pi}{2})$
$R_x(\frac{\pi}{2})$
detect |1⟩
Displacement
creation
tomography

Even and odd superpositions

Squeezing Trisqueezing Quadsqueezing



S. Saner et al., arXiv:2409.03482 (2024)

Conclusion

- Demonstrated new protocol for spin-mediated bosonic interactions

- First demonstration of fourth-order interactions

- Incorporated mid-circuit measurements to generate arbitrary superposition

- Future work
  - Extend to other nonlinear interactions
  - More modes
  - More complete characterisation of these states

arXiv:2507.19588 [pdf, ps, other] quant-ph

Real-Time Observation of Aharonov-Bohm Interference in a Z_2 Lattice Gauge Theory on a Hybrid Qubit-Oscillator Quantum Computer

Authors: S. Saner, O. Băzăvan, D. J. Webb, G. Araneda, C. J. Ballance, R. Srinivas, D. M. Lucas, A. Bermúdez

Abstract: Quantum simulations of lattice gauge theories (LGTs) with both dynamical matter and gauge fields provide a promising approach to studying strongly coupled problems beyond classical computational reach. Yet, implementing gauge-invariant encodings and real-time evolution remains experimentally challenging. Here, we demonstrate a resource-efficient encoding of a Z_2 LGT using a hybrid qubi… ▽ More

Submitted 25 July, 2025; **originally announced** July 2025.

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Sebastian Saner

Oana Băzăvan

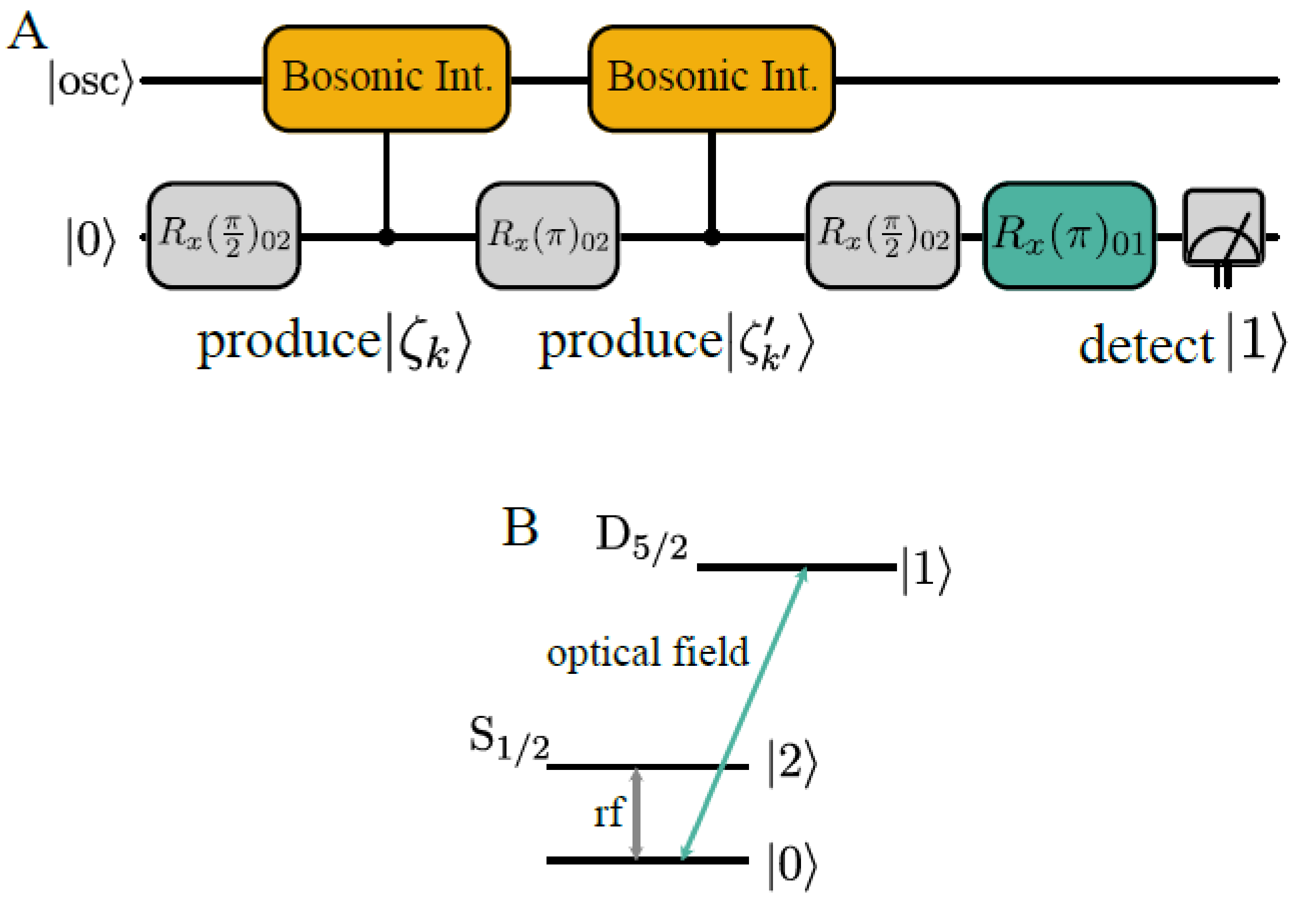
Peter Drmota

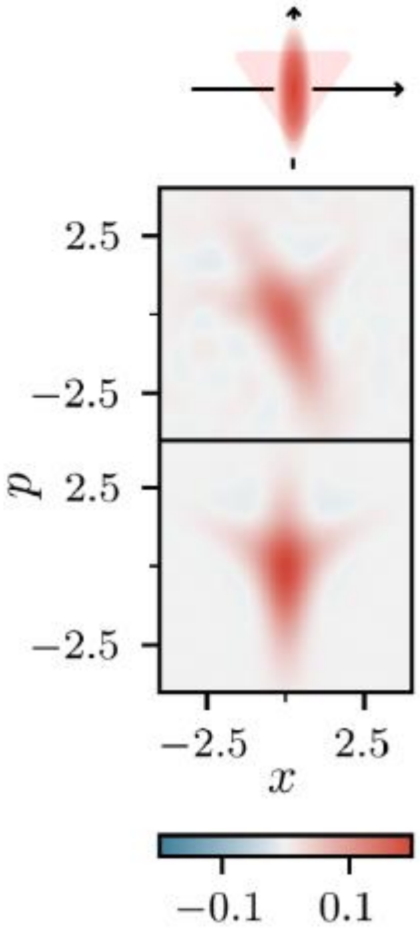
Gabriel Araneda

Donovan Webb

David Nadlinger

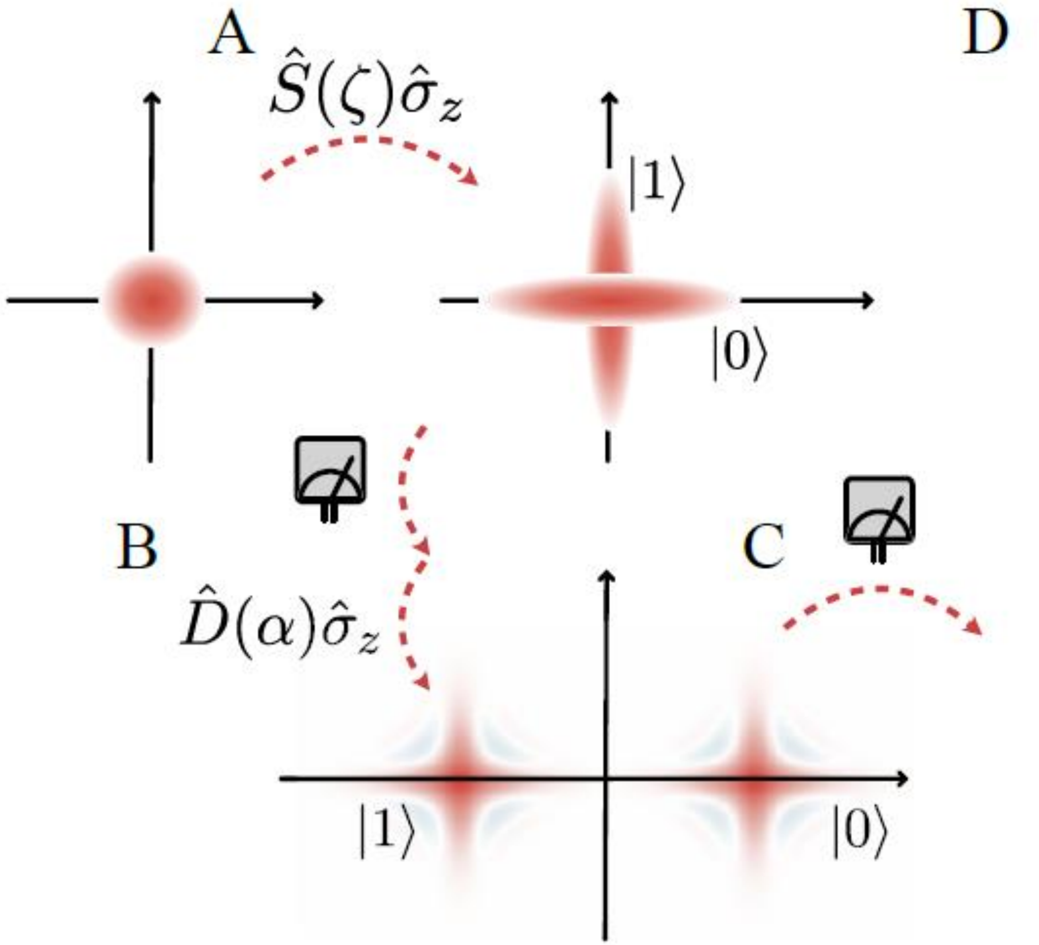
Superpositions of different states





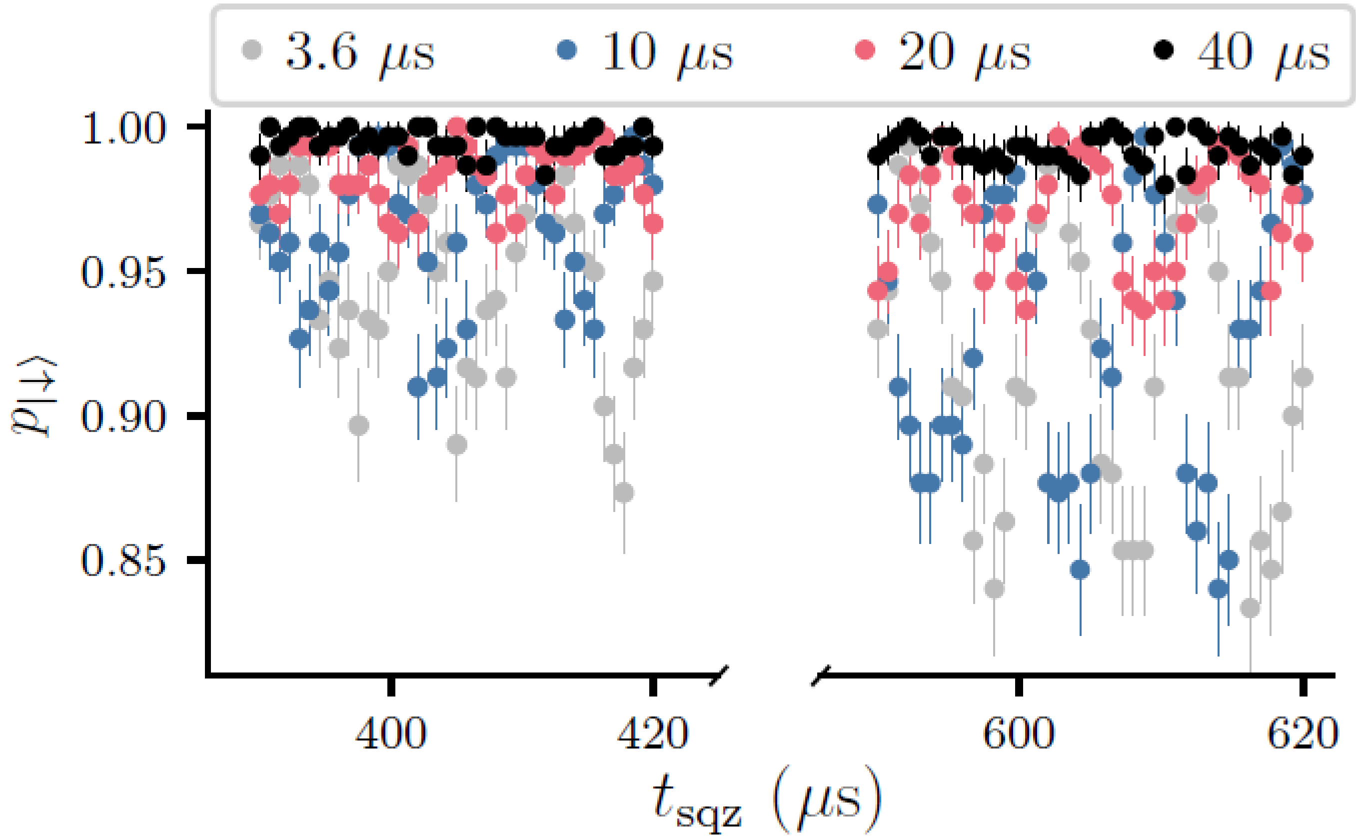
S. Saner et al., arXiv:2409.03482 (2024)

Superpositions of displaced states



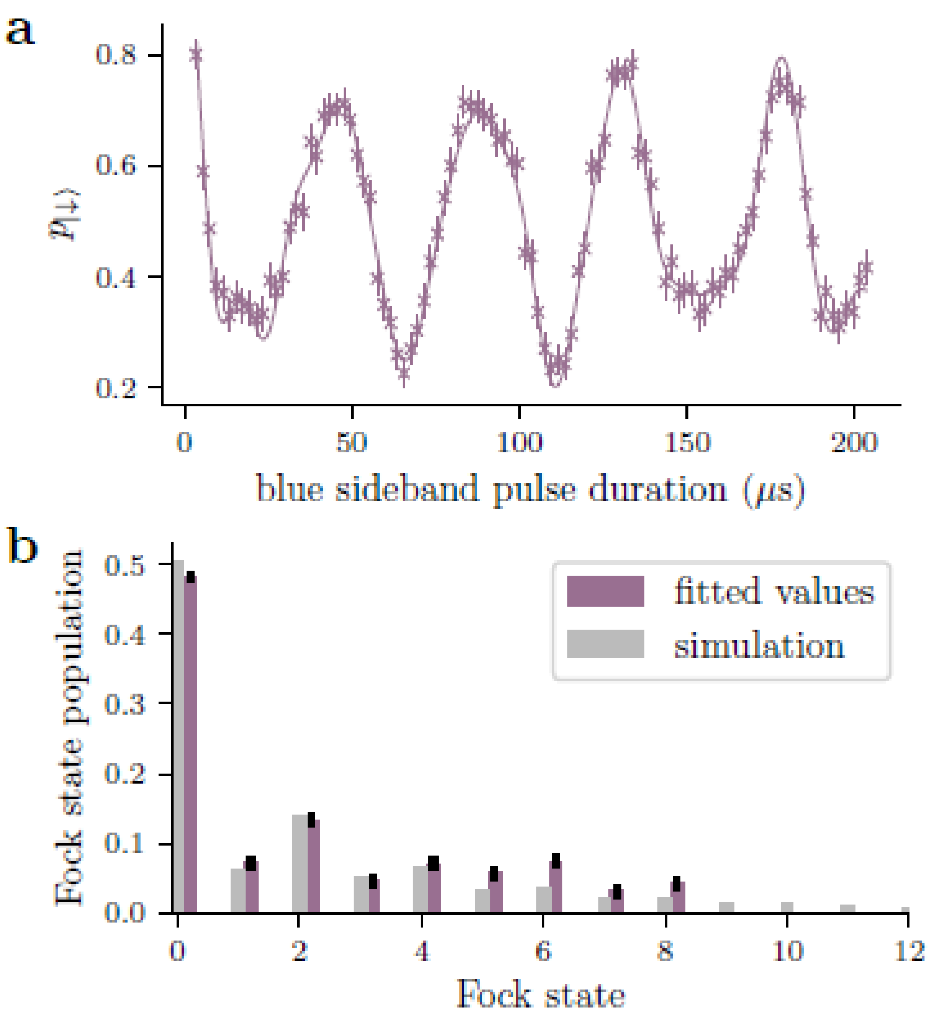
S. Saner et al., arXiv:2409.03482 (2024)

Ramps

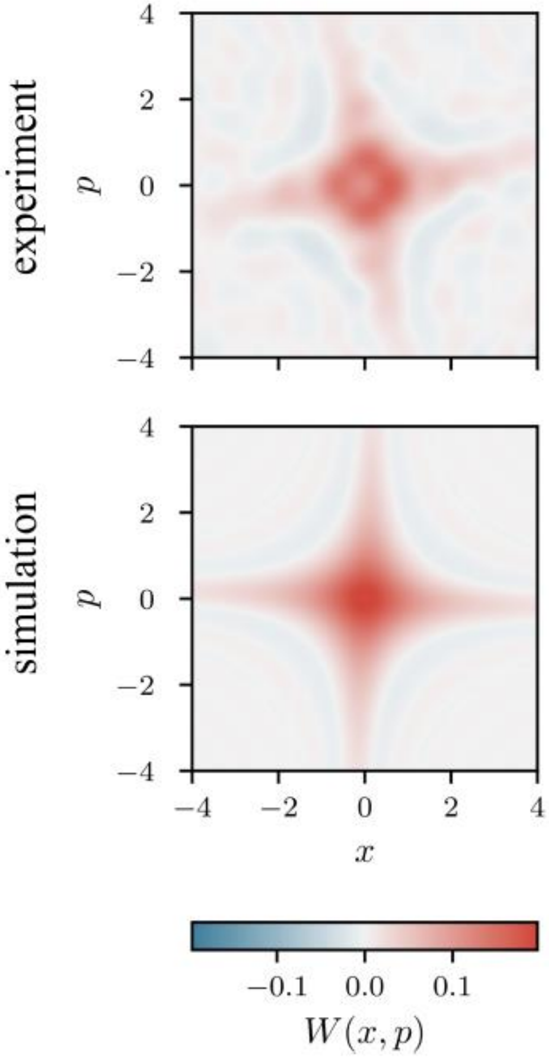


3.6 μs
10 μs
20 μs
40 μs
1.00
0.95
0.90
0.85
$p_{|\downarrow\rangle}$
400
420
600
620
t_{sqz} (μs)

Fock state analysis



Wigner negativity



Rabi frequency scaling

$$\Omega_{2,3,4} = \left\{ \frac{\Omega_{\alpha'}\Omega_{\alpha}}{\Delta}, \frac{\Omega_{\alpha'}\Omega_{\alpha}^{2}}{2\Delta^{2}}, \frac{\Omega_{\alpha'}\Omega_{\alpha}^{3}}{8\Delta^{3}} \right\}$$

$$\Omega_{\eta^2,\eta^3,\eta^4} = \left\{ \frac{\Omega_c \eta^2}{2!}, \frac{\Omega_c \eta^3}{3!}, \frac{\Omega_c \eta^4}{4!} \right\}$$