Non-Hermitian Evolution from Continuous Monitoring

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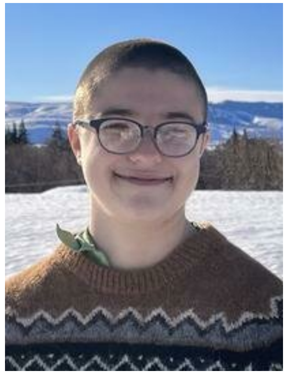
Outline

- **Superconducting Qubits**
  - Circuit QED Overview
  - Dispersive Readout and Trajectories

- **Modeling Measurement**
  - Reduced Qubit: Steady-State Scattering
  - Qubit-Resonator: Non-Hermitian Evolution



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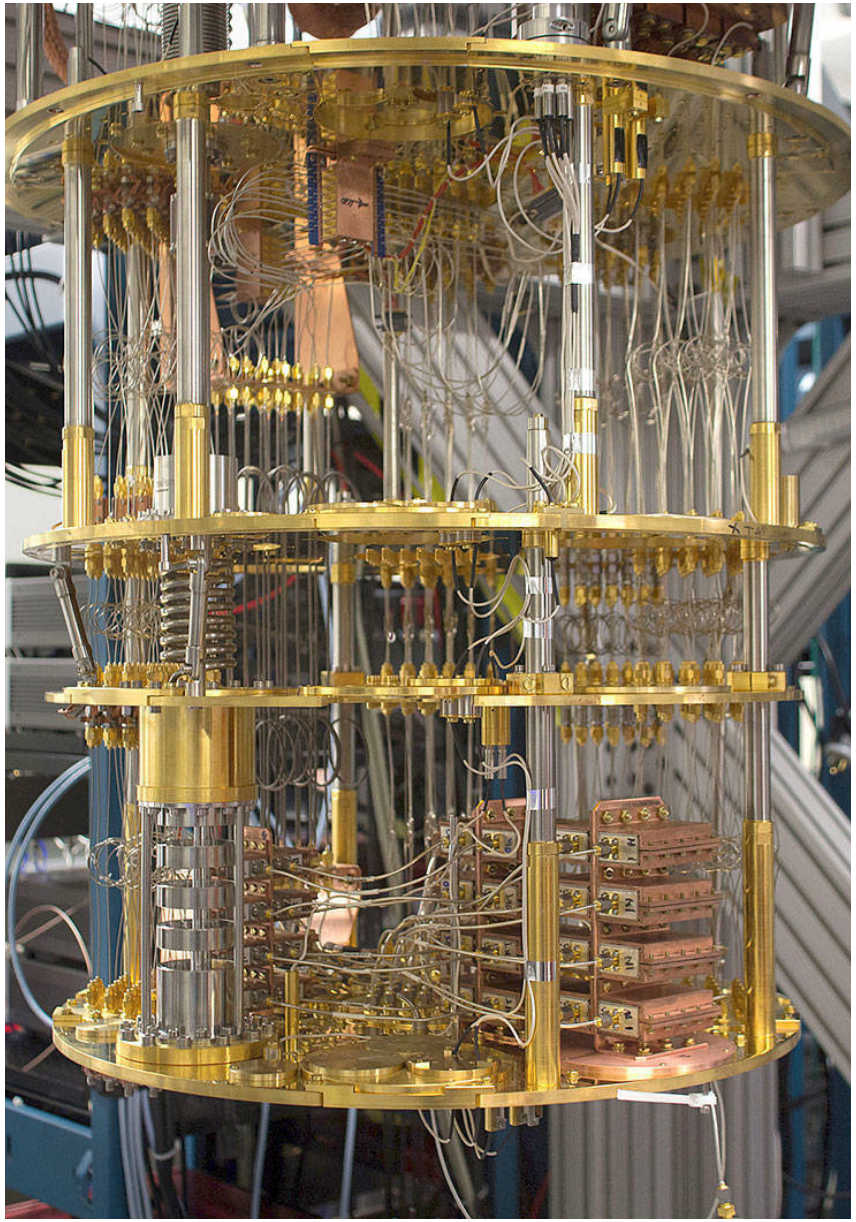
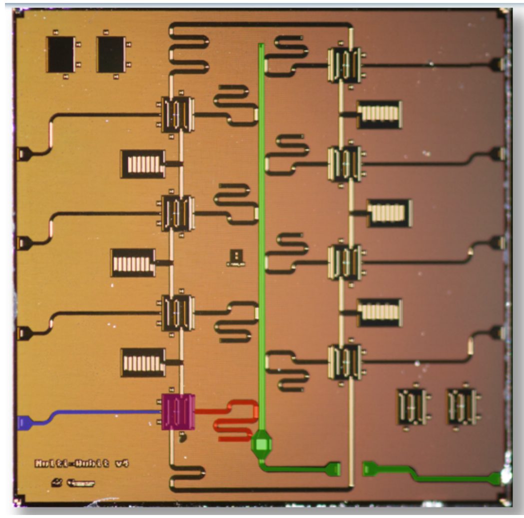
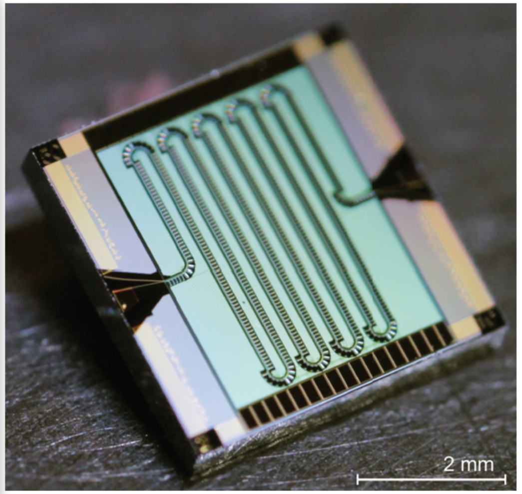
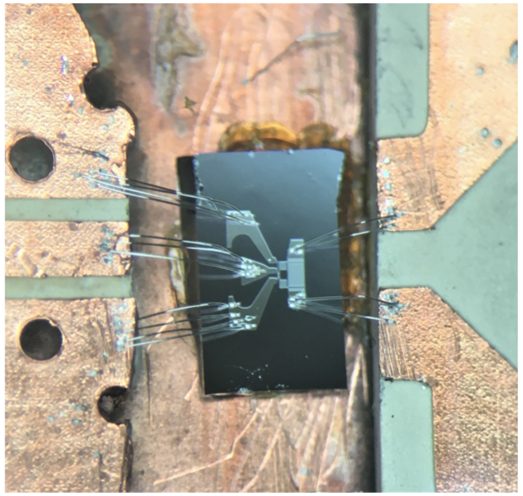


Superconducting Qubits







Mesoscopic coherence of **collective charge motion** at μm scale, mK temperature

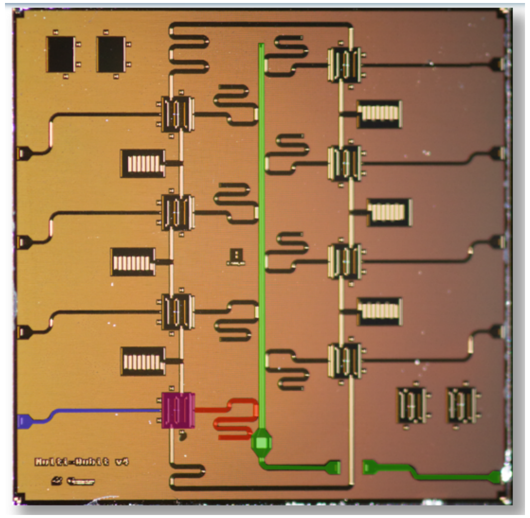
EM Fields of charge motion described by **Circuit QED**

Anharmonic oscillator potentials treated as **artificial atoms,** with lowest 2 levels as **qubit**

Qubit levels **controlled** by resonant **microwave field** drives

Qubit levels **measured** via dispersive frequency-shift of coupled **microwave resonator** mode

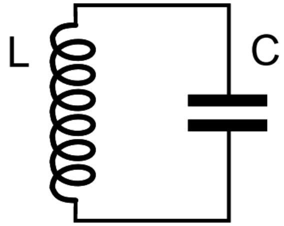
Circuit QED Overview



Canonically conjugate dynamical variables: $[\hat{\Phi}, \hat{Q}] = i\hbar$

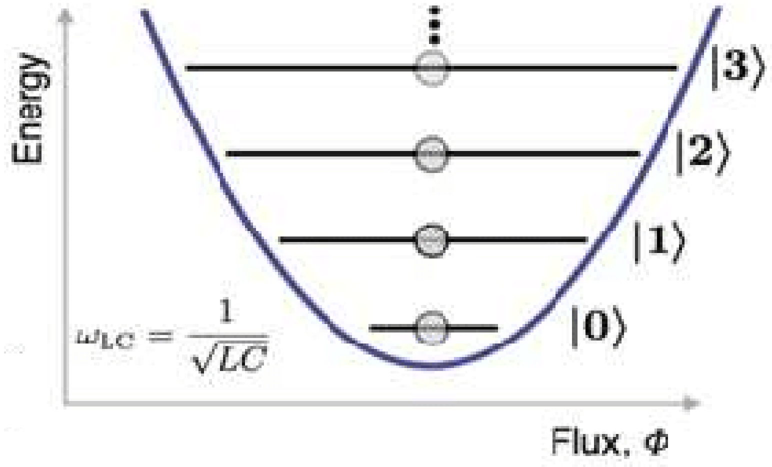
- inductive (magnetic) **flux**: $\hat{\Phi} = \Phi_0\, \hat{\phi}$, $\Phi_0 = \hbar/2e$
- capacitive (electric) **charge**: $\hat{Q} = (2e)\, \hat{n}$

Dimensionless conjugate variables: $[\hat{\phi},\, \hat{n}] = i$

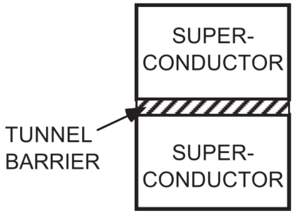


Example: Harmonic Oscillator

Useful circuit, but **bad qubit**:
Can't isolate specific level pairs since
all energy gaps are identical



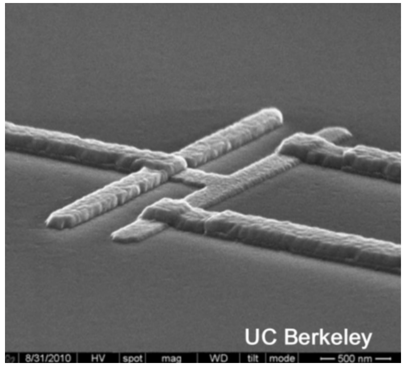
Quantum Pendulum

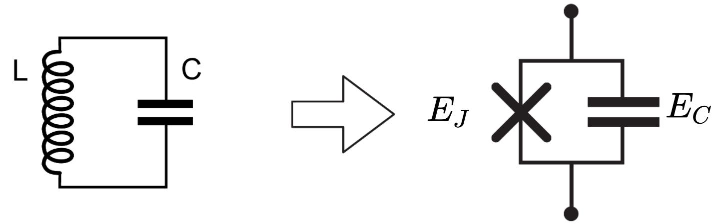


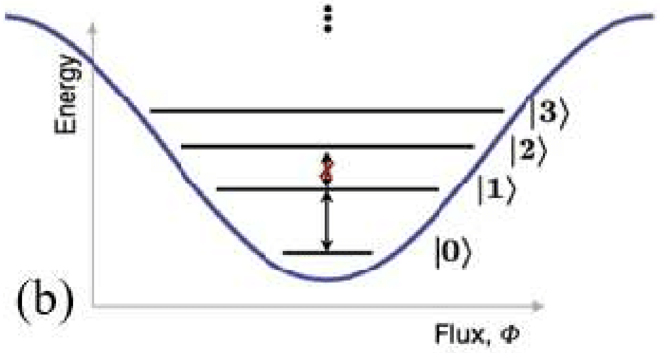
Josephson Junction: $\hat{H}_J = E_J\left(1 - \cos\hat{\phi}\right) \approx \frac{E_J}{2}\,\hat{\phi}^2 - \frac{E_J}{4!}\hat{\phi}^4 + \cdots$

$\hat{\phi} = \frac{\hat{\Phi}}{\Phi_0}, \quad E_J = \frac{\Phi_0^2}{L_J}$ Acts as *nonlinear inductance* => *anharmonic oscillator*

Shunting with large capacitor shields from charge noise





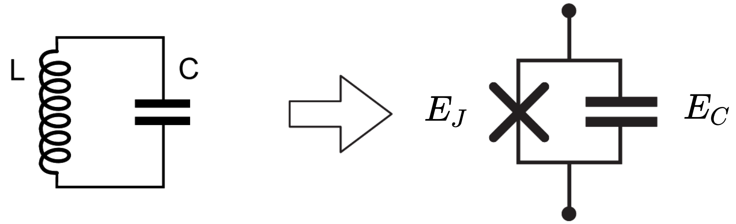


Nonlinear inductance makes energy gaps different
Energy level pairs now addressable as qubits

Multiple levels are bound in the cosine well, like an **artifical atom**
The bottom two levels are the most stable qubit

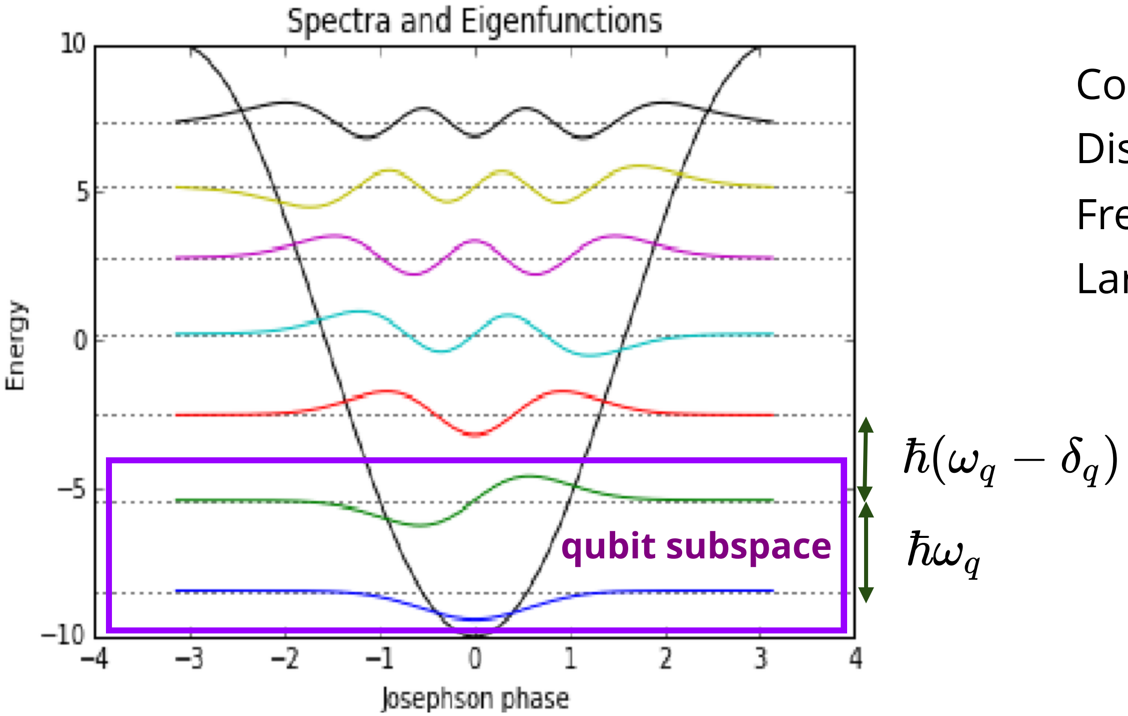
Microwave drive resonant with qubit energy gap
induces single-qubit gates (controlled Rabi oscillations)

Transmon Qubit



$$\hat{H} = \hat{H}_C + \hat{H}_J = E_C\,(2\hat{n})^2 + E_J\,(1 - \cos\hat{\phi})$$

Large shunt capacitor $\dfrac{E_J}{E_C} \approx 100$



$\omega_q/2\pi \sim 4\text{–}7\text{GHz}$ $\delta_q/2\pi \sim 100\text{–}300\text{MHz}$

Cosine potential acts like an **artificial atom** with ~7 levels

Distinct level spacings allow **targeted control** of specific pairs of levels

Frequency gaps in the **microwave** regime

Large capacitor **protects against charge noise**

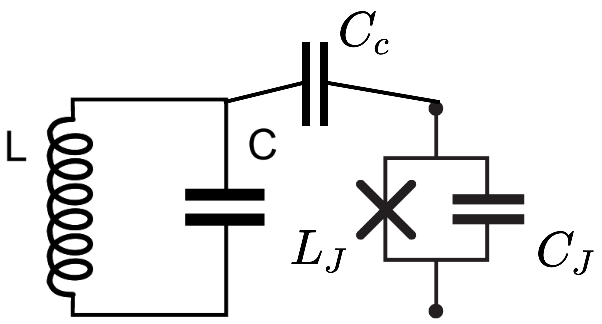
Distinct behavior from optical regime with real atoms:

- Engineered chips permit *ultra-strong* and *deep-strong* coupling regimes that are difficult to achieve with atoms
- Lower frequencies than optics make *transients* more relevant
- *Emission* can be *directionally controlled* down waveguides to minimize collection loss and increase detection efficiency

$$\hat{H} \approx E_0\,\hat{1} + \hbar\omega_q\,|1\rangle\langle 1| + \hbar(2\omega_q - \delta_q)\,|2\rangle\langle 2| \approx \frac{E_0}{2}\hat{1} - \frac{\hbar\omega_q}{2}\hat{\sigma}_z$$

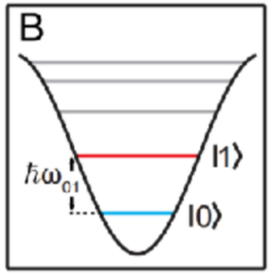
$\hat{\sigma}_z = |0\rangle\langle 0| - |1\rangle\langle 1|$ **Qubit Pauli Z**

How do we Measure a Superconducting Qubit?



Problem:

Qubit is on a chip inside a fridge near absolute zero.
How does one "measure the energy eigenstate" of the qubit *without causing unwanted changes to those energy states*?



Transmon qubit

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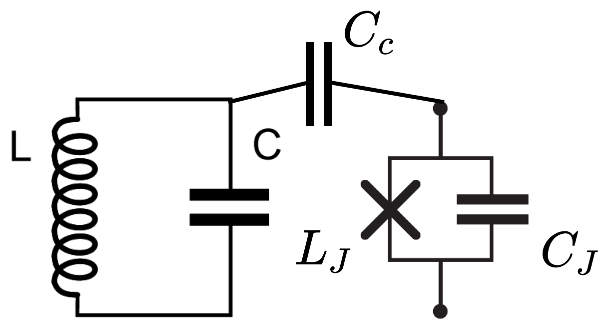

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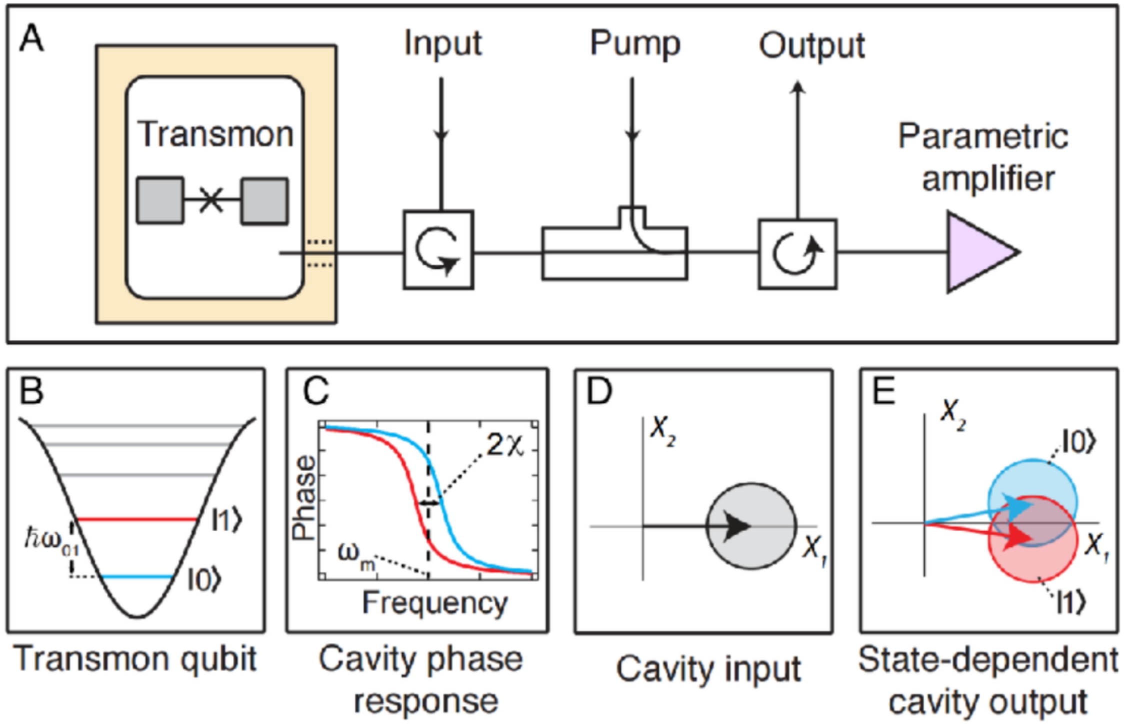


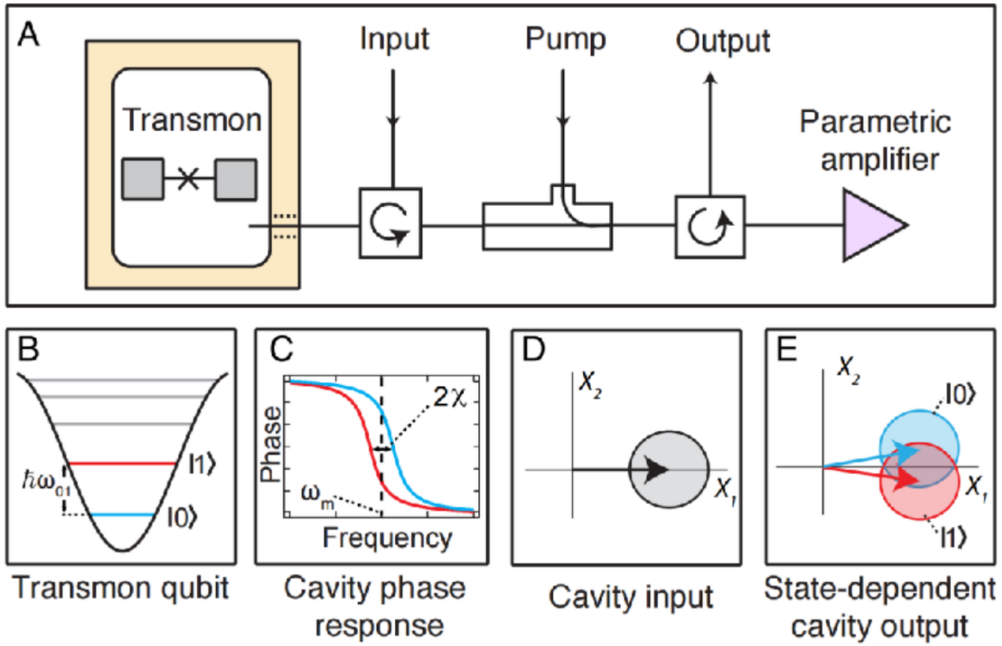
Solution:

Indirectly peek at the qubit energy by *dispersively coupling* the qubit to a *strongly detuned resonator*, then probing that resonator with a microwave tone.

The *frequency shift* of the leaked and amplified tone stores *information* about the energy state of the qubit *without allowing energy transitions* between the qubit and resonator.

This type of measurement that does not disturb energy eigenstates in the process of measuring the energy is called a *"quantum non-demolition (QND) measurement"*.

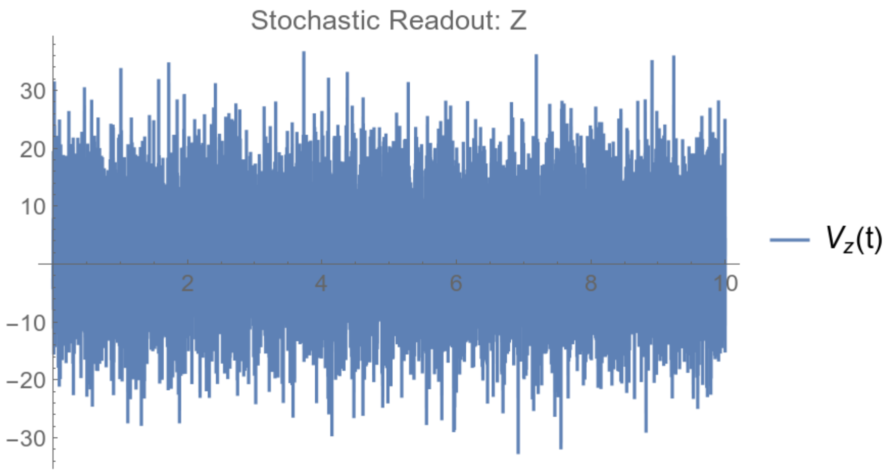
Dispersive Readout: Quantum Filtering



Problem:

The microwave probe tone must be *compared* to the original source, using *homodyne* or *heterodyne* measurements.

These signals are noisy due to the intrinsic vacuum noise of the source, which dominates the tiny amount of qubit information per unit time.



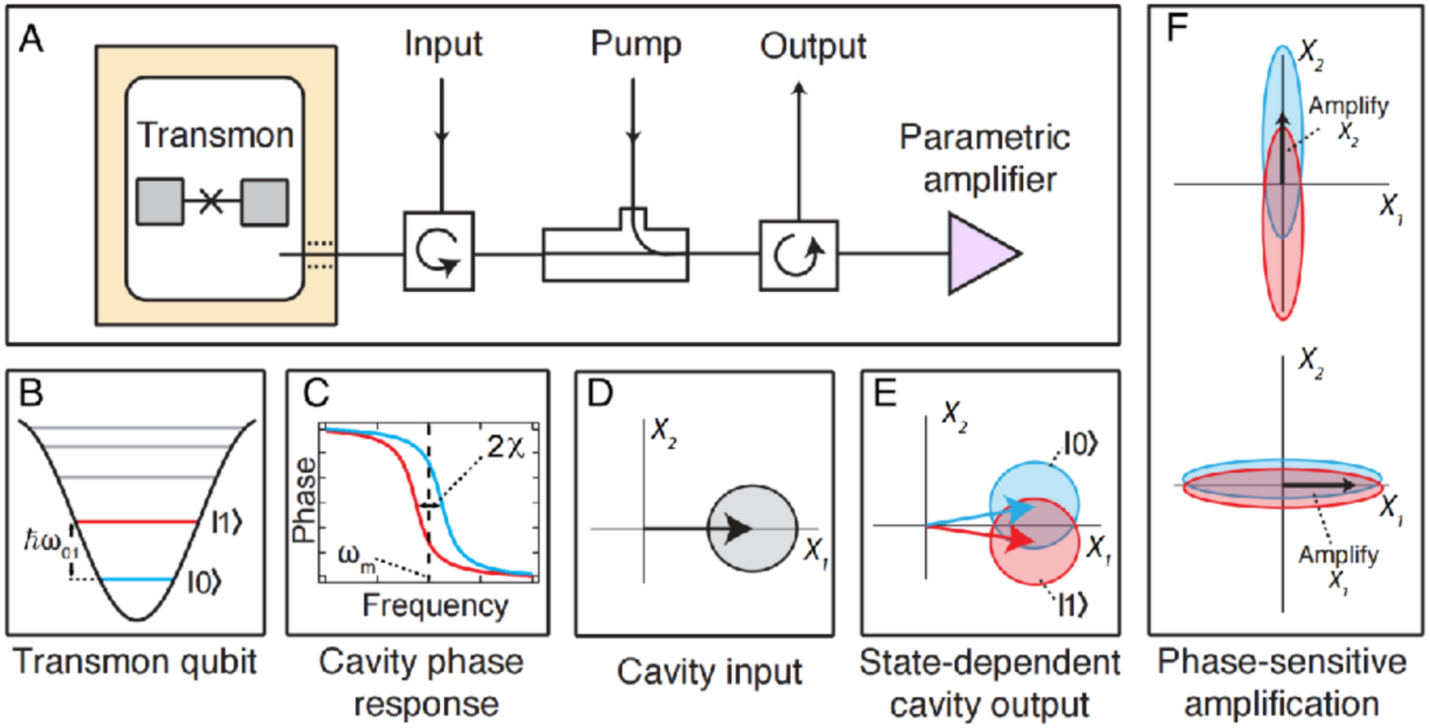
Need to:

- process *stochastic* voltage records (**classical signal filtering**)
- infer corresponding state evolution (**quantum state filtering**)

Bayes' Rule from probability theory and the

Born rule from quantum mechanics dictate how
the **state partially collapses** with information gain!

Dispersive Readout: Amplification



Outgoing signal is **further amplified** to enhance phase difference in steady-state resonator modes

One (*informational*) quadrature encodes the **qubit state information** as a displacement of the signal distribution

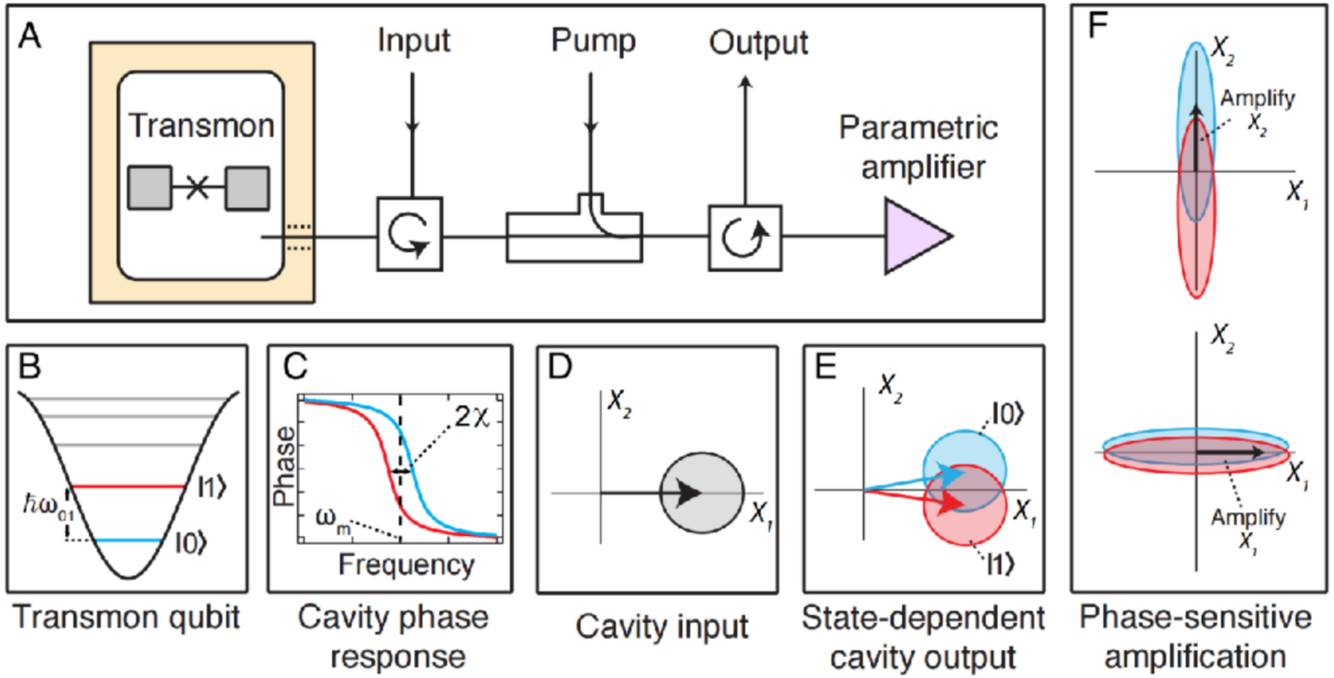
The orthogonal (*phase*) quadrature encodes **photon number fluctuations** inside the resonator

Quantum-limited amplifiers (built from Josephson junctions using their nonlinear inductance):

- **Josephson Parametric Amplifier (JPA)**: narrow-band reflected 3-wave mixer
  - can operate in phase-sensitive (squeezed, above) or phase-preserving (unsqueezed) modes

- **Traveling Wave Parametric Amplifier (TWPA)**: *broad-band* amplification over transmission-line propagation
  - only operates in phase-preserving (unsqueezed) mode

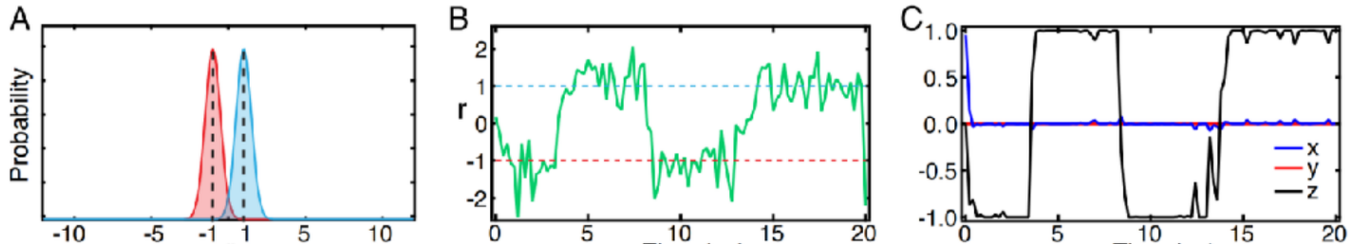
Strong Continuous Monitoring



Stronger coupling yields more *distinguishable* resonator states

More information per unit time yields more *rapid projection* to the stationary eigenstates of the coupling

=> **Strong continuous measurement**

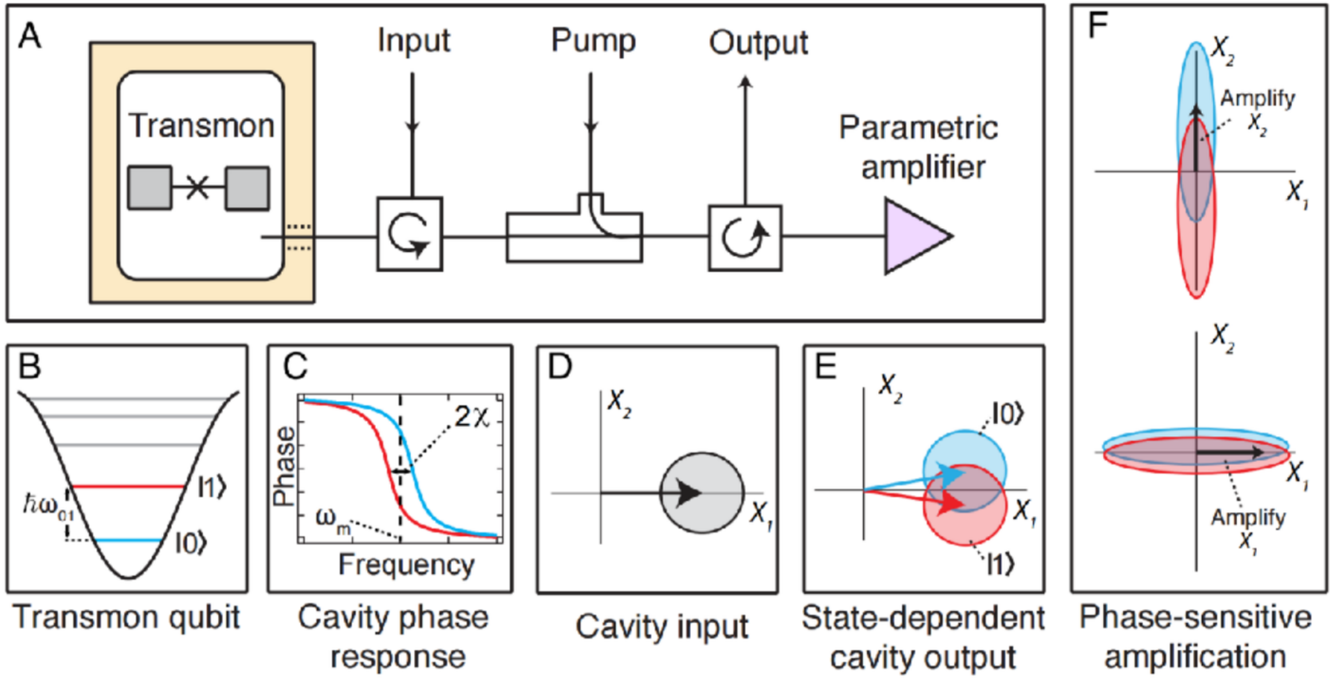


Single shot "projective" readout

- Integrated signal clearly distinguishes definite qubit states
- "quantum jumps" visible (useful for syndrome detection)
- Prevents normal dynamics from occurring (*quantum Zeno effect*)

arXiv:1506.08165

Weak Continuous Monitoring



Weaker coupling yields
less distinguishable resonator states

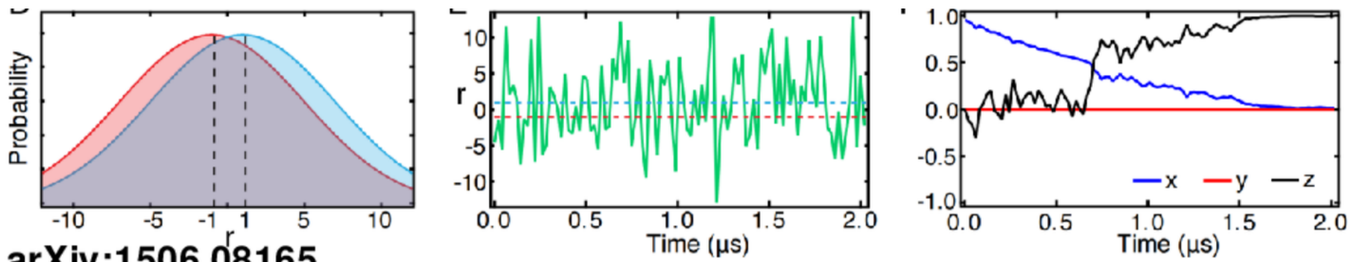
Less information per unit time yields
slower projection to the coupling eigenstates

=> **Weak continuous measurement**

Following Bayes' rule, the qubit state *randomly walks* as probabilities are updated with each chunk of information in the signal

- Quantum state less affected per unit time
- **Same average information** collected over an ensemble
- Can "gently" **monitor** average information **during dynamics,**
- State **gradually collapses** to an eigenstate as information accumulates.

$$P(k|r) = \frac{P(r|k)P(k)}{P(r|1)P(1) + P(r|0)P(0)}$$



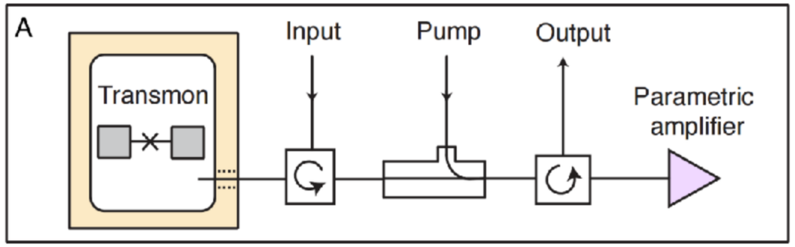
arXiv:1506.08165

$$|\psi\rangle = \sqrt{P(0)}|0\rangle + \sqrt{P(1)}e^{i\phi}|1\rangle$$
$$|\psi\rangle \xrightarrow{r} \sqrt{P(0|r)}|0\rangle + \sqrt{P(1|r)}e^{i\phi}|1\rangle$$

Steady-state Scattering Model

- **Transmon + Resonator + Transmission Line**:

Koroktov, Phys. Rev. A 94, 042326 (2016)
JD group, Phys. Rev. A 96, 022311 (2017)



$|\psi\rangle_r$ $|\psi\rangle_{dx}$ $|\psi\rangle_{2dx}$ $\cdots$

$|\psi\rangle_q$

$$dx = v\,dt$$

$$\begin{aligned} |\Psi(2dt)\rangle &= c_0|0\rangle|\alpha_0(2dt)\rangle|\sqrt{\kappa dt}\,\alpha_0(dt)\rangle\cdots \\ &+ c_1|1\rangle|\alpha_1(2dt)\rangle|\sqrt{\kappa dt}\,\alpha_1(dt)\rangle\cdots \end{aligned}$$

Entanglement stores **memory** of past interactions

- Coherent drive yields 2 qubit-dependent **resonator coherent states** : $|\alpha_0\rangle$, and $|\alpha_1\rangle$
- Beam-splitter-like **scattering** to transmission line
- Per unit time dt, amplitude $\sqrt{\kappa dt}$ of coherent field leaks into transmission line segment of width dx
- Once in tail, each dx simply propagates down the line from segment to segment

- Tracing out the transmission line yields effective "dephasing"/"decoherence" of the qubit state
- Measuring the transmission line **collapses** the entangled factors and changes qubit amplitudes, yielding stochastic dynamics

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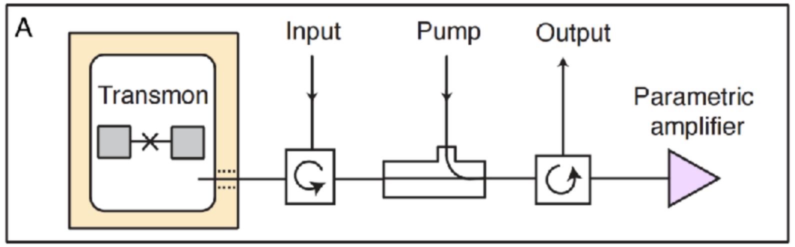

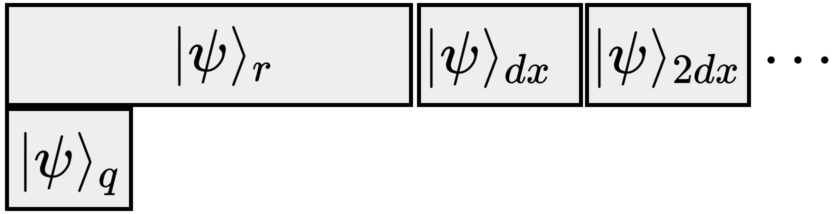

$dx = vdt$

- Resonator evolution: $\dfrac{d\alpha_{0/1}}{dt} = \mp i\chi\alpha_{0/1} - \dfrac{\kappa}{2}\alpha_{0/1} - i\varepsilon$

- Coherent steady states: $\alpha_{0/1} = \dfrac{-i2\varepsilon}{\kappa}\dfrac{1}{1 \pm i2\chi/\kappa}$, $\qquad \bar{n} = |\alpha_{0/1}|^2 = \dfrac{4|\varepsilon|^2}{\kappa^2}\dfrac{1}{1+(2\chi/\kappa)^2}$

- Reduced qubit-resonator state:

$$\hat{\rho}_{qr} = |c_0|^2\,|0\rangle\langle 0| \otimes |\alpha_0\rangle\langle\alpha_0| + |c_1|^2\,|1\rangle\langle 1| \otimes |\alpha_1\rangle\langle\alpha_1| + c_0 c_1^*\,\langle\sqrt{\kappa dt}\,\alpha_1|\sqrt{\kappa dt}\,\alpha_0\rangle\;|0\rangle\langle 1| \otimes |\alpha_0\rangle\langle\alpha_1| + \mathrm{h.c.}$$

- Measurement-dephasing rate and ac-Stark shift: $\langle\sqrt{\kappa dt}\,\alpha_1|\sqrt{\kappa dt}\,\alpha_0\rangle = \exp(-\Gamma_m dt + i\,\omega_S dt)$

$$\Gamma_m = \frac{\kappa|\alpha_1-\alpha_0|^2}{2} = \frac{8\chi^2\bar{n}}{\kappa(1+(2\chi/\kappa)^2)}, \qquad \omega_S = \kappa\,\mathrm{Im}\alpha_1^*\alpha_0 = \frac{4\chi\bar{n}}{1+(2\chi/\kappa)^2}$$

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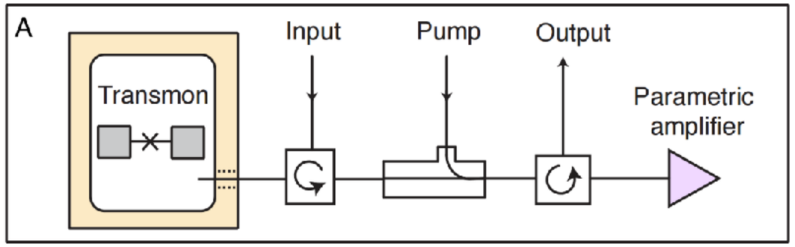

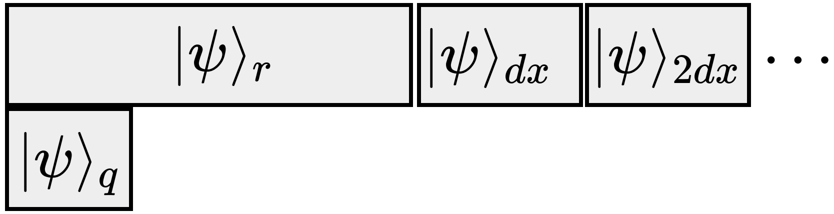

$$dx = vdt$$

- Post-measurement qubit-resonator state after measuring quadrature $\langle I_\theta|$:

$$|\Psi_{qr}\rangle' = c_0\ \langle I_\theta|\sqrt{\kappa dt}\,\alpha_0\rangle\ |0\rangle\,|\alpha_0\rangle + c_1\ \langle I_\theta|\sqrt{\kappa dt}\,\alpha_1\rangle\ |1\rangle\,|\alpha_1\rangle = (\hat{M}_{I_\theta}\otimes\hat{1})\,|\Psi_{qr}\rangle$$

$$\langle I_\theta|\sqrt{\kappa dt}\,\alpha_{0/1}\rangle = \frac{1}{\pi}\exp\left[-\frac{1}{2}(I_\theta - \sqrt{2\kappa dt}\,\mathrm{Re}(\alpha_{0/1}e^{i\theta}))^2 + I_\theta\sqrt{2\kappa dt}\,\mathrm{Im}(\alpha_{0/1}e^{i\theta}) - i\kappa dt\,\mathrm{Re}(\alpha_{0/1}e^{i\theta})\mathrm{Im}(\alpha_{0/1}e^{i\theta})\right]$$

- Qubit Gaussian POVM:

$$\hat{M}^\dagger_{I_\theta}\hat{M}_{I_\theta}dI_\theta = \hat{M}^\dagger_{r,\theta}\hat{M}_{r,\theta}dr = \sqrt{\frac{\Gamma_m dt}{\pi}}\exp\left(-dt\Gamma_m(r-\cos\theta\,\hat{\sigma}_z)^2\right)dr,$$

$$r = \frac{I_\theta}{\sqrt{\Gamma_m dt}} - \frac{\kappa}{2\chi}\sin\theta$$

- Qubit Measurement (Kraus) Operator:

$$\hat{M}_{r,\theta} = (\Gamma dt)^{1/4}[\langle I_\theta|\sqrt{\kappa dt}\,\alpha_0\rangle\,|0\rangle\langle 0| + \langle I_\theta|\sqrt{\kappa dt}\,\alpha_1\rangle\,|1\rangle\langle 1|] = \sqrt{\bar{p}(r,\theta)}\,\hat{U}_\theta\ \exp(dt\,\Gamma_m r e^{-i\theta}\hat{\sigma}_z)$$

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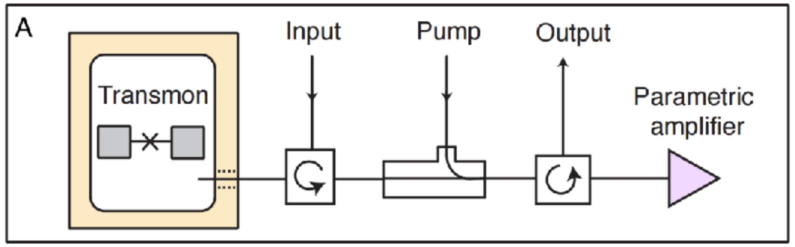
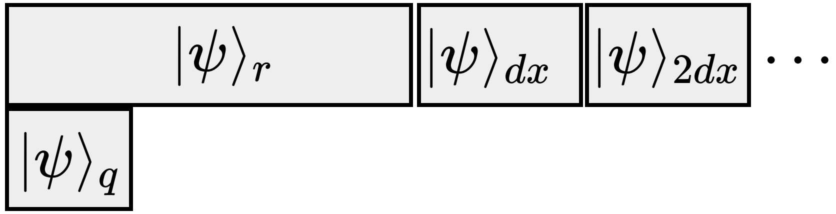




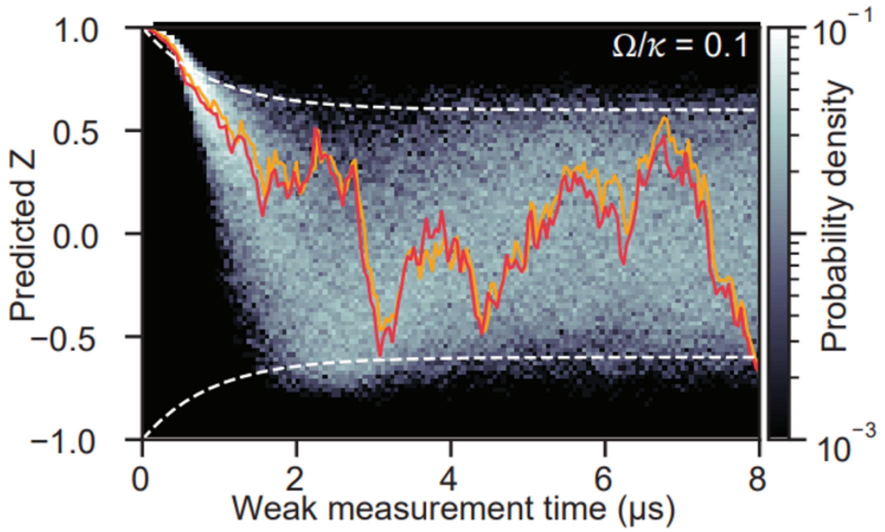
$$dx = vdt$$

- **Take-aways:**
  - Steady **coherent-state scattering model works extremely well** to predict experiment
  - Depends on **transmission line "boxcar"** of width $dx = vdt$, set by the **detector time resolution**
  - Each boxcar in the transmission line train is an **independent Hilbert space factor**
  - Each boxcar must be an **commuting bosonic mode** supporting compatible coherent states

- **Limitations:**
  - Assumes resonator returns to steady state faster than each boxcar time dt: **no transient evolution**!
  - Assumes qubit evolution is adiabatic on relaxation timescale: **no fast qubit dynamics!**
  - Assumes conditional coherent states: **no exotic resonator states**!
  - Good for analytic qubit formulation: **no numerical simulation** of resonator dynamics!

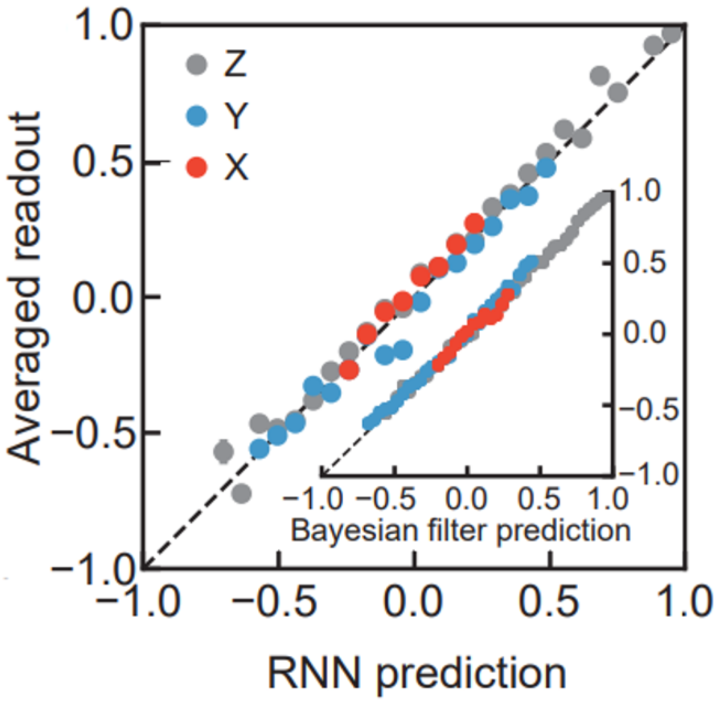
Machine Learning of Quantum Trajectories

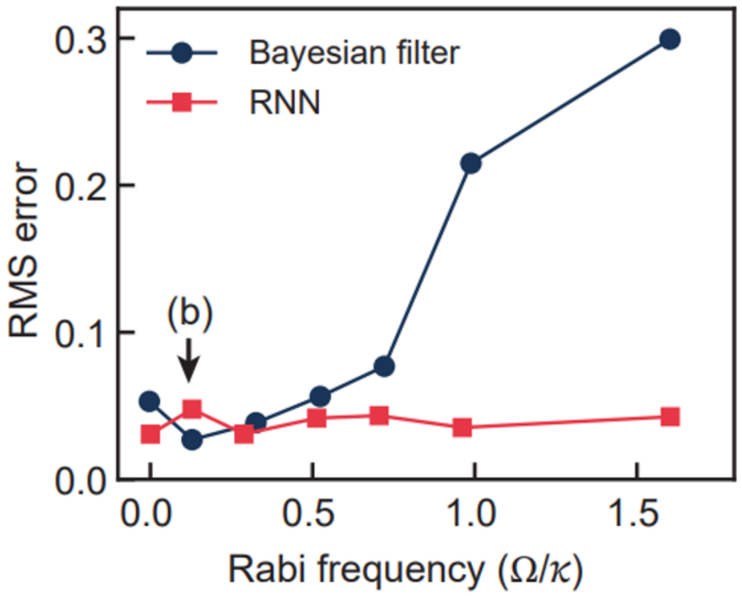
Siddiqi group, JD, *PRX* 12, 031017 (2022)



- For **slow** Rabi drive ($2\Omega_R/\kappa = 0.2$):
  - RNN trajectories (red) agree well with standard Bayesian update trajectories (orange)
  - Measurement dephasing from inefficiency balances purification over Rabi period to confine diffusive trajectories

- RNN trajectory predictions compare favorably to *tomographic strong readout* (for the same slow Rabi trajectories shown on left)
- Bayesian comparison inset





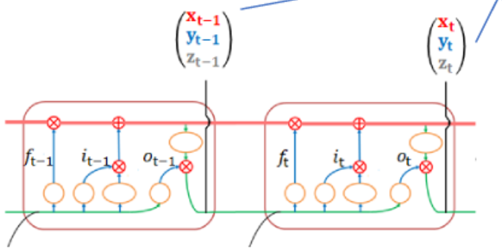
- RMS error averaged over the three qubit coordinates shown above vs. Rabi frequency
- For **faster** Rabi drive ($2\Omega_R/\kappa \geq 1$) *the simple Bayesian update filter fails* while the ***LSTM RNN filter maintains consistent accuracy***

Machine Learning of Quantum Trajectories

Siddiqi group, JD, *PRX* 12, 031017 (2022)

LSTM RNN produces successive state estimates, so directly predicts the **state increment** at **every pair of adjacent time bins**

$$d\rho_{t-1} = \rho_t - \rho_{t-1}$$

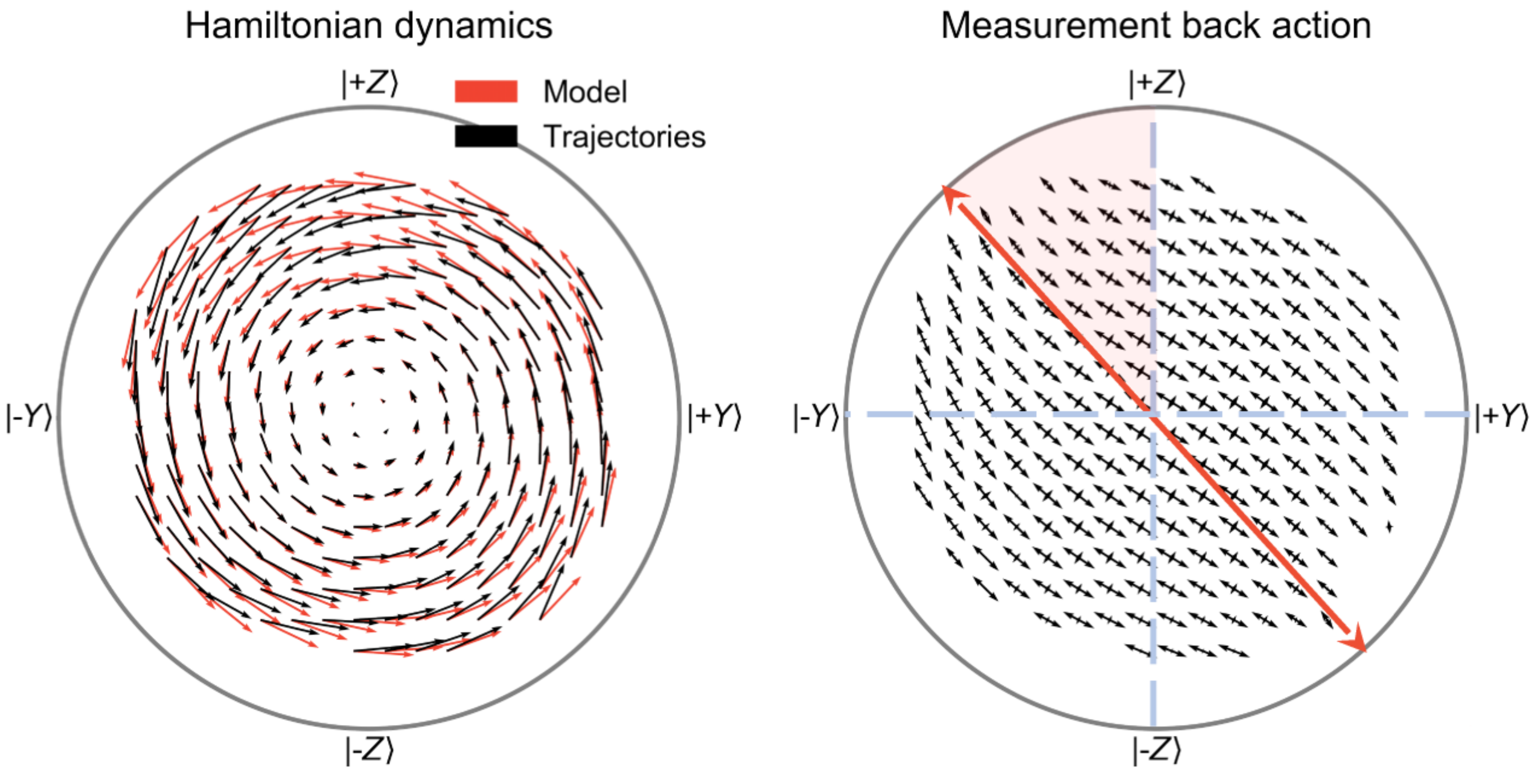


State update parameters can be fit and extracted, and their dynamical contributions analyzed independently

$$d\rho_t = \underbrace{\left(i[H_R, \rho_t] + \mathcal{L}\left[\sqrt{\frac{\gamma_\phi}{2}}\sigma_Z\right]\rho_t\right)dt}_{\text{deterministic evolution}}$$

$$+ \underbrace{\sqrt{\eta}\mathcal{H}\left[\sqrt{\frac{\gamma_\phi}{2}}\sigma_Z\right]\rho_t dw_t}_{\text{backaction}},$$

$$2\Omega_R/\kappa \sim 1.4$$

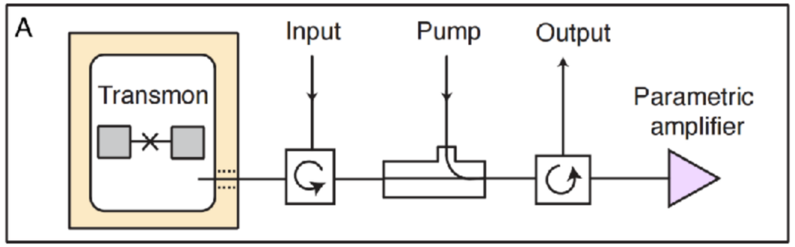


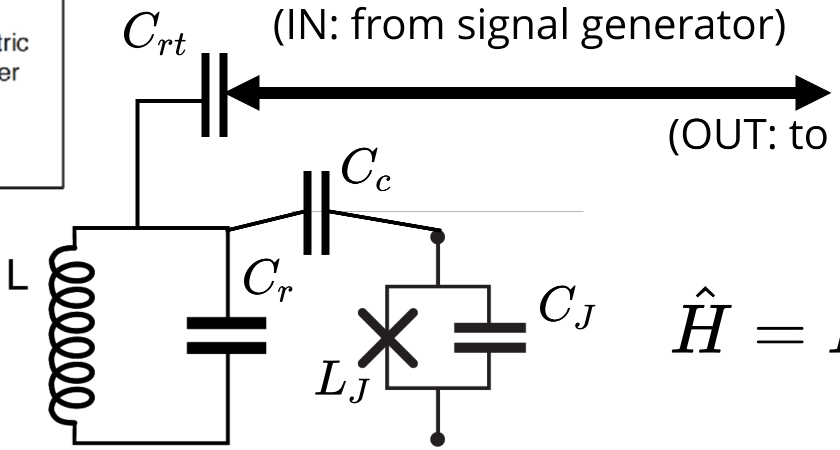
Extracted **RNN Hamiltonian dynamics agrees well** with that expected from Rabi control, even with faster drives

Extracted **RNN collapse dynamics reveals Rabi-induced physics**:
1. Tilted measurement axis
2. Reduced collapse rate

Circuit-QED: Transmission-line Fields

Campagne-Ibarcq (2017)





c, l Capacitance, Inductance per unit length

$v = \frac{1}{\sqrt{lc}} \qquad Z_t = \sqrt{\frac{l}{c}}$

$$\hat{H} = \hat{H}_q + \hat{H}_r + \hat{H}_c + \hat{H}_t + \hat{H}_{rt}$$

$$\hat{H}_t = \int_0^\lambda \left[\frac{\hat{q}(x)^2}{2c} + \frac{(\partial_x \hat{\phi}(x))^2}{2l} \right] dx = v \int_0^\lambda (\hat{A}^{\leftarrow}(x)^2 + \hat{A}^{\rightarrow}(x)^2) dx = \sum_{k=-\infty}^{\infty} \hbar\omega_k \frac{\hat{c}_k^\dagger \hat{c}_k + \hat{c}_k \hat{c}_k^\dagger}{2}$$

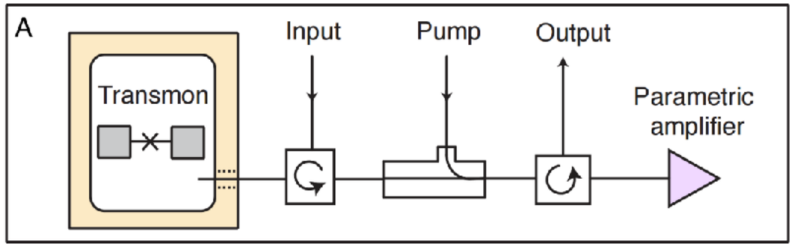
Transmission line: ***local quantum fields*** $\quad [\hat{\phi}(x), \hat{q}(x')] = i\hbar\delta(x - x')$

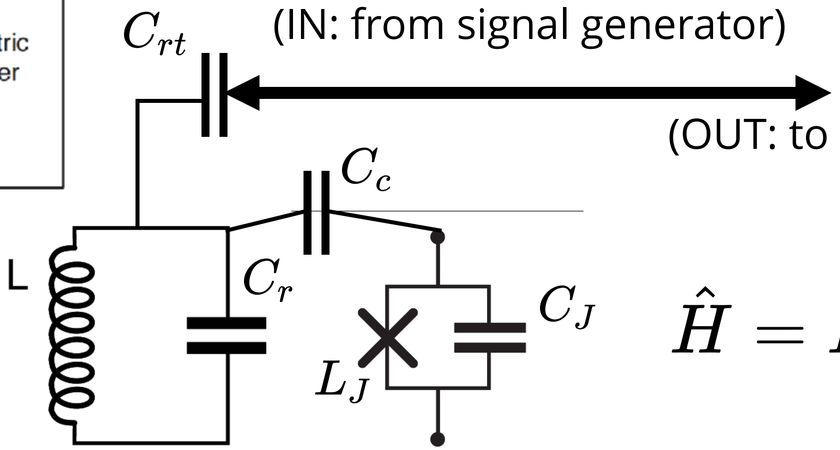
Local traveling waves: $\quad \hat{A}^{\rightleftharpoons}(s = t \mp x/v) = \frac{1}{2\sqrt{v}} \left(\frac{\partial_s \hat{\phi}(s)}{\sqrt{l}} \pm \frac{\hat{q}(s)}{\sqrt{c}} \right) \qquad [\hat{A}^{\rightleftharpoons}(s), \hat{A}^{\rightleftharpoons}(s')] = \frac{i\hbar}{2} \partial_s \delta(s - s')$

Global harmonic modes: $\quad \hat{c}_k = \sqrt{\frac{2v}{\lambda}} \int_0^\lambda \hat{A}^{\rightarrow}(x) \frac{e^{-ikx}}{\sqrt{\hbar\omega_k}} dx, \; k > 0 \qquad [\hat{c}_k, \hat{c}_{k'}^\dagger] = \delta_{kk'}$

Measured bosonic modes are local wavelet packets

Resonator-Transmission-line Coupling





$$\hat{H} = \hat{H}_q + \hat{H}_r + \hat{H}_c + \hat{H}_t + \hat{H}_{rt}$$

$$\begin{aligned}
\hat{H}_{rt} &= \hat{\Phi}\hat{I}(x=0) \\
&= \sqrt{\frac{\hbar Z_r}{2}}(\hat{a}+\hat{a}^\dagger)\sqrt{\frac{1}{Z_t}}(\hat{A}^{\rightarrow}(t) - \hat{A}^{\leftarrow}(t)) \\
&= \frac{\hbar}{2}\sqrt{\frac{Z_r}{Z_t}}(\hat{a}+\hat{a}^\dagger)\sqrt{\frac{v}{\lambda}}\sum_{k=0}^{\infty}\sqrt{\omega_k}(\hat{c}_{-k} + \hat{c}_{-k}^\dagger - \hat{c}_k - \hat{c}_k^\dagger) \\
&\equiv \frac{\hbar}{2} i\sqrt{\kappa}(\hat{a}+\hat{a}^\dagger)(\hat{c}_{\text{in}} - \hat{c}_{\text{in}}^\dagger - \hat{c}_{\text{out}} + \hat{c}_{\text{out}}^\dagger)
\end{aligned}$$

$$\kappa \equiv \omega_r \frac{Z_r}{Z_t}$$ **Resonator decay rate near ω_r**

$$\hat{c}_{\text{in}} \equiv -i\sqrt{\frac{v}{\lambda}}\sum_{k=0}^{\infty}\sqrt{\frac{\omega_k}{\omega_r}}\,\hat{c}_{-k}$$

$$\hat{c}_{\text{out}} \equiv -i\sqrt{\frac{v}{\lambda}}\sum_{k=0}^{\infty}\sqrt{\frac{\omega_k}{\omega_r}}\,\hat{c}_{k}$$

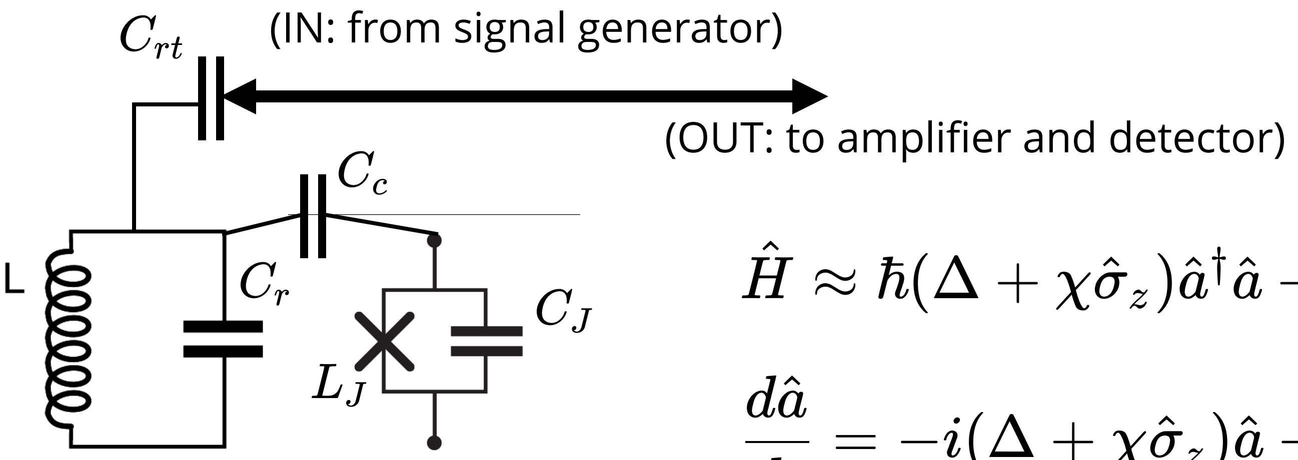
Approximately white in narrow frequency band near ω_r

Input-output (boundary) condition:

$$\hat{\Phi} = \hat{\phi}(x=0)$$

$$\Rightarrow \sqrt{\kappa}\,\hat{a}e^{-i\omega_d t} = (\hat{c}_{\text{in}} + \hat{c}_{\text{out}})e^{-i\omega_d t}$$ **(RWA)**

Effective RWA Resonator Evolution



$$\hat{H} \approx \hbar(\Delta + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \frac{\hbar}{2} i\sqrt{\kappa}[\hat{a}^\dagger(\hat{c}_{\text{in}} - \hat{c}_{\text{out}}) - \hat{a}(\hat{c}^\dagger_{\text{in}} - \hat{c}^\dagger_{\text{out}})]$$

$$\frac{d\hat{a}}{dt} = -i(\Delta + \chi\hat{\sigma}_z)\hat{a} + \frac{\sqrt{\kappa}}{2}(\hat{c}_{\text{in}} - \hat{c}_{\text{out}})$$

$$\Delta = \omega_r - \omega_d$$

$$\sqrt{\kappa}\,\hat{a}e^{-i\omega_d t} = (\hat{c}_{\text{in}} + \hat{c}_{\text{out}})e^{-i\omega_d t}$$

$$\hat{c}_{\text{in}}(t) \approx -i(\varepsilon(t)/\sqrt{\kappa} + \hat{v}(t))$$

Vacuum fluctuations

$$[\hat{v}(t), \hat{v}^\dagger(t')] = \delta(t - t')$$

Markovian vacuum white noise

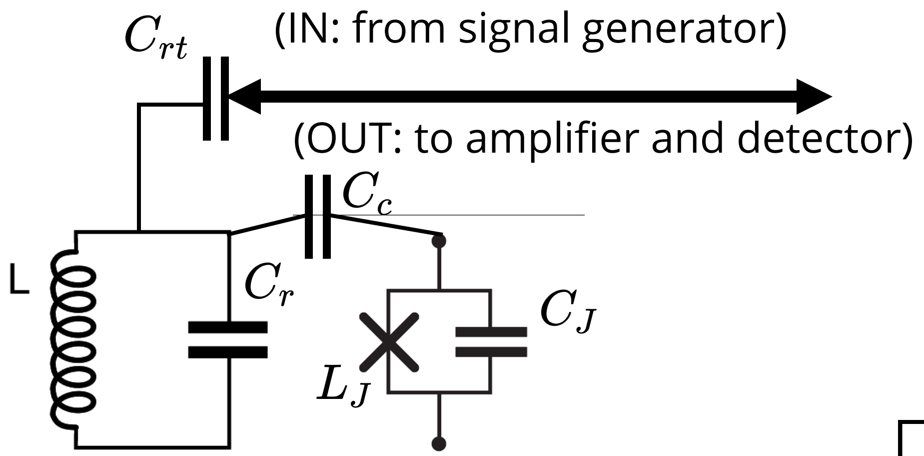
Quantum Langevin Equation (RWA):

$$\boxed{\frac{d\hat{a}}{dt} = -i(\Delta + \chi\hat{\sigma}_z)\hat{a} - \frac{\kappa}{2}\hat{a} - i(\varepsilon(t) + \sqrt{\kappa}\hat{v}(t))}$$

Reflected field in transmission line:

$$\boxed{\hat{c}_{\text{out}}(t) = \sqrt{\kappa}\,\hat{a}(t) + i(\varepsilon(t)/\sqrt{\kappa} + \hat{v}(t))}$$

Effective Boxcar Propagator



Traveling reflected field:

$$\hat{c}_{\text{out}}(t) = \sqrt{\kappa}\,\hat{a}(t) + i(\varepsilon(t)/\sqrt{\kappa} + \hat{v}(t))$$

Finite bandwidth detector absorbs demodulated ***compact wavelet mode***:

$$\boxed{\hat{b}_{\text{out}}(t) = \int_{t-\Delta t}^{t} \hat{c}_{\text{out}}(t')\,\frac{dt'}{\sqrt{\Delta t}} \approx \sqrt{\kappa\Delta t}\,\hat{a}(t) - \hat{b}_{\text{in}}(t)}$$

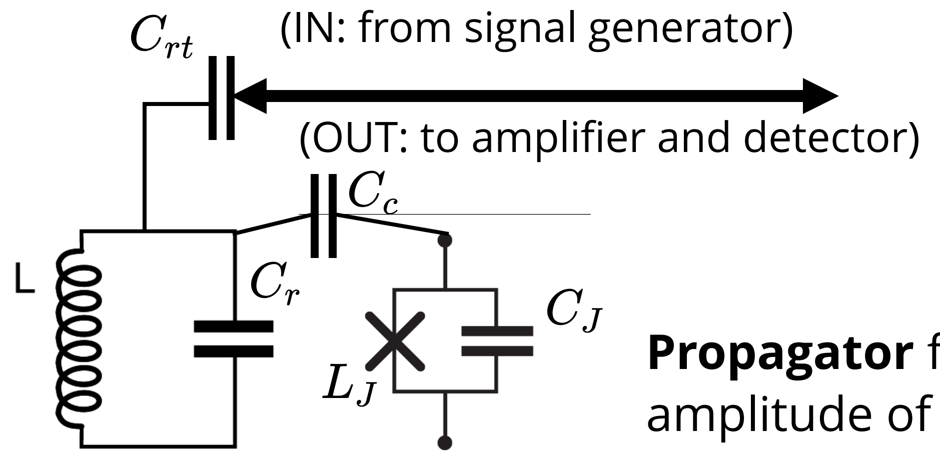
Bosonic commutator of wavelet mode must be preserved:

$$\boxed{\langle\hat{b}_{\text{in}}(t)\rangle \approx -i\varepsilon(t')\sqrt{\frac{\Delta t}{\kappa}}, \qquad [\hat{b}_{\text{in}}(t), \hat{b}_{\text{in}}^{\dagger}(t)] = 1 \qquad \Longrightarrow \qquad [\hat{b}_{\text{out}}(t), \hat{b}_{\text{out}}^{\dagger}(t)] = 1}$$

Propagator for digitized evolution "boxcar" duration Δt:

$$\boxed{\hat{U} = \exp\left[-\frac{i}{\hbar}\int_{t-\Delta t}^{t} \hat{H}(t')dt'\right] = \exp\left[-i\Delta t(\Delta + \chi\hat{\sigma}_z)\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\sqrt{\kappa\Delta t}[\hat{a}^{\dagger}(\hat{b}_{\text{in}} - \hat{b}_{\text{out}}) - \hat{a}(\hat{b}_{\text{in}}^{\dagger} - \hat{b}_{\text{out}}^{\dagger})]\right]}$$

Effective Boxcar Displacement and Initial State



$$\boxed{\hat{b}_{\text{out}}(t) = \sqrt{\kappa\Delta t}\,\hat{a}(t) - \hat{b}_{\text{in}}(t)}$$

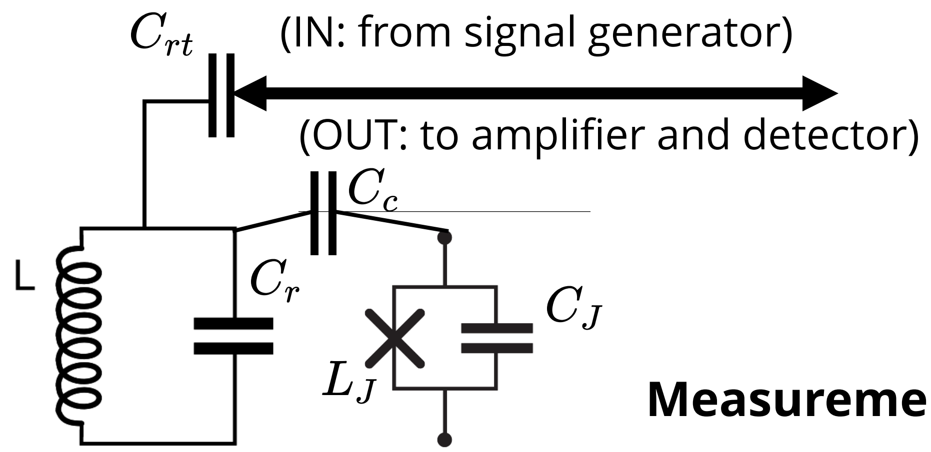
Propagator for "boxcar" looks like effective *displacement* of *output* mode by leaked amplitude of the resonator field!

$$\begin{aligned}\hat{U} &= \exp\left[-i\Delta t(\Delta + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \frac{1}{2}\sqrt{\kappa\Delta t}[\hat{a}^\dagger(\hat{b}_{\text{in}} - \hat{b}_{\text{out}}) - \hat{a}(\hat{b}^\dagger_{\text{in}} - \hat{b}^\dagger_{\text{out}})]\right] \\ &= \exp\left[-i\Delta t(\Delta + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \boxed{\hat{b}^\dagger_{\text{out}}(\sqrt{\kappa\Delta t}\,\hat{a}) - \hat{b}_{\text{out}}(\sqrt{\kappa\Delta t}\,\hat{a}^\dagger)}\right]\end{aligned}$$

Initial coherent state for *input* mode is equivalent to **resonator drive Hamiltonian** and equivalent choice of **initial *negated* coherent state for the *output* mode**:

$$\begin{aligned}|-i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\text{in}} &= \exp\left[\hat{b}^\dagger_{\text{in}}(-i\varepsilon\sqrt{\Delta t/\kappa}) - \hat{b}_{\text{in}}(i\varepsilon^*\sqrt{\Delta t/\kappa})\right]|0\rangle \\ &= \boxed{\exp\left[-i\Delta t(\hat{a}^\dagger\varepsilon + \hat{a}\varepsilon^*)\right]|+i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\text{out}}}\end{aligned}$$

Return of the Steady-State Picture



$$\hat{U} = \exp\left[-i\Delta t(\Delta + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \hat{b}^\dagger_{\rm out}(\sqrt{\kappa\Delta t}\,\hat{a}) - \hat{b}_{\rm out}(\sqrt{\kappa\Delta t}\,\hat{a}^\dagger)\right]$$

$$|-i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\rm in} = \exp\left[-i\Delta t(\hat{a}^\dagger\varepsilon + \hat{a}\varepsilon^*)\right]|+i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\rm out}$$

Measurement of output mode **post-selects** a particular state $\langle I_\theta|$, yielding an effective evolution **Kraus operator**:

$$\boxed{\begin{aligned}\hat{M}_{I_\theta} &= {}_{\rm out}\langle I_\theta|\,\hat{U}\,|-i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\rm in} \\ &= {}_{\rm out}\langle I_\theta|\exp\left[-i\Delta t(\Delta + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \hat{b}^\dagger_{\rm out}(\sqrt{\kappa\Delta t}\,\hat{a}) - \hat{b}_{\rm out}(\sqrt{\kappa\Delta t}\,\hat{a}^\dagger)\right]\exp\left[-i\Delta t(\hat{a}^\dagger\varepsilon + \hat{a}\varepsilon^*)\right]|+i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\rm out}\end{aligned}}$$

When the **resonator** is in a **steady coherent state** $|\alpha_{0/1}\rangle$, this simplifies:

$$\hat{M}_{I_\theta} \to \exp\left[-i\Delta t((\Delta + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \hat{a}^\dagger\varepsilon + \hat{a}\varepsilon^*)\right]\,{}_{\rm out}\langle I_\theta| + i\varepsilon\sqrt{\Delta t/\kappa} + \sqrt{\kappa\Delta t}\alpha_{0/1}\rangle_{\rm out}$$

Resonator Hamiltonian evolution

After subtraction of background reflected input pump, *this is the expected coherent state overlap of the steady-state boxcar picture!*

Non-Hermitian Hamiltonian Evolution

Phase-preserving measurement of output mode **post-selects** a particular coherent state: $\langle +i\varepsilon\sqrt{\Delta t/\kappa} + \sqrt{\kappa\Delta t}\, r|$

This choice subtracts the reflected pump and scales the readout result r to match the resonator $\hat{a}$

$$\begin{aligned}\hat{M}_r &= {}_{\text{out}}\langle +i\varepsilon\sqrt{\Delta t/\kappa} + \sqrt{\kappa\Delta t}\, r|\, \hat{U}\, |-i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\text{in}} \\ &= e^{-i\Delta t(\Delta+\chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} - i\Delta t(\hat{a}^\dagger\varepsilon + \hat{a}\varepsilon^*)}\, {}_{\text{out}}\langle +i\varepsilon\sqrt{\Delta t/\kappa} + \sqrt{\kappa\Delta t}\, r|\, e^{\hat{b}^\dagger_{\text{out}}(\sqrt{\kappa\Delta t}\,\hat{a}) - \hat{b}_{\text{out}}(\sqrt{\kappa\Delta t}\,\hat{a}^\dagger)}\, |+i\varepsilon\sqrt{\Delta t/\kappa}\rangle_{\text{out}}\end{aligned}$$

Expanding to linear order in Δt yields effectively **Non-Hermitian Hamiltonian evolution**:

$$\boxed{\begin{aligned}&\hat{M}_r = \exp\left(-\frac{i\Delta t}{\hbar}\hat{H}_{\text{eff}}(r)\right)\, {}_{\text{out}}\langle \sqrt{\kappa\Delta t}\, r|0\rangle_{\text{out}} \\ &\hat{H}_{\text{eff}}(r) = \hbar(\Delta + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \hbar(\hat{a}^\dagger\varepsilon + \hat{a}\varepsilon^*) + i\hbar\kappa r^*\,\hat{a} - i\hbar\kappa\hat{a}^\dagger\hat{a}/2\end{aligned}}$$

Stochastic measurement backaction
(depends on random result r)

"No-jump" **Lindblad resonator decay**

An Alternative Approach: Husimi Q

$$dP(r|\hat{\rho}) = \mathrm{Tr}(d\hat{P}(r)\,\hat{\rho}) = \iint dP_Q(r|\alpha)\,P(\alpha|\hat{\rho})\,d^2\alpha,$$

$$dP_Q(r|\alpha) = \frac{d^2r}{\pi}\sqrt{\frac{\kappa\Delta t}{\pi}}\;\exp\left[-\kappa\Delta t|r-\alpha|^2\right] = \frac{\langle\alpha|\,d\hat{P}(r)\,|\alpha\rangle}{\pi}$$

The Q-representation encodes eigenvalues α for normally-ordered field-operator products.

The POVM $d\hat{P}(r)$ can be evaluated and factored into a pair of *Kraus operators* $\hat{M}_r$ and $\hat{N}$

$$\boxed{\begin{aligned} d\hat{P}(r) &\equiv d^2r\,p_0(r)\;\exp(\kappa\Delta t\,r\hat{a}^\dagger)[1-\kappa\Delta t]^{\hat{a}^\dagger\hat{a}}\,\exp(\kappa\Delta t\,r^*\hat{a}) \equiv d^2r\,p_0(r)\,(\hat{N}\hat{M}_r)^\dagger\,(\hat{N}\hat{M}_r),\\ p_0(r) &= \sqrt{\frac{\kappa\Delta t}{\pi}}\;\exp(-\kappa\Delta t\,|r|^2) \propto |\,\langle\sqrt{\kappa\Delta t}\,r|0\rangle\,|^2,\\ \hat{M}_r &\equiv \exp(\kappa\Delta t\,r^*\hat{a}),\\ \hat{N} &\equiv [1-\kappa\Delta t]^{\hat{a}^\dagger\hat{a}/2} \approx \exp(-\kappa\Delta t\hat{a}^\dagger\hat{a}/2) \end{aligned}}$$

Phase-preserving Measurements

Regime 1:

$2\chi/\kappa = 1$

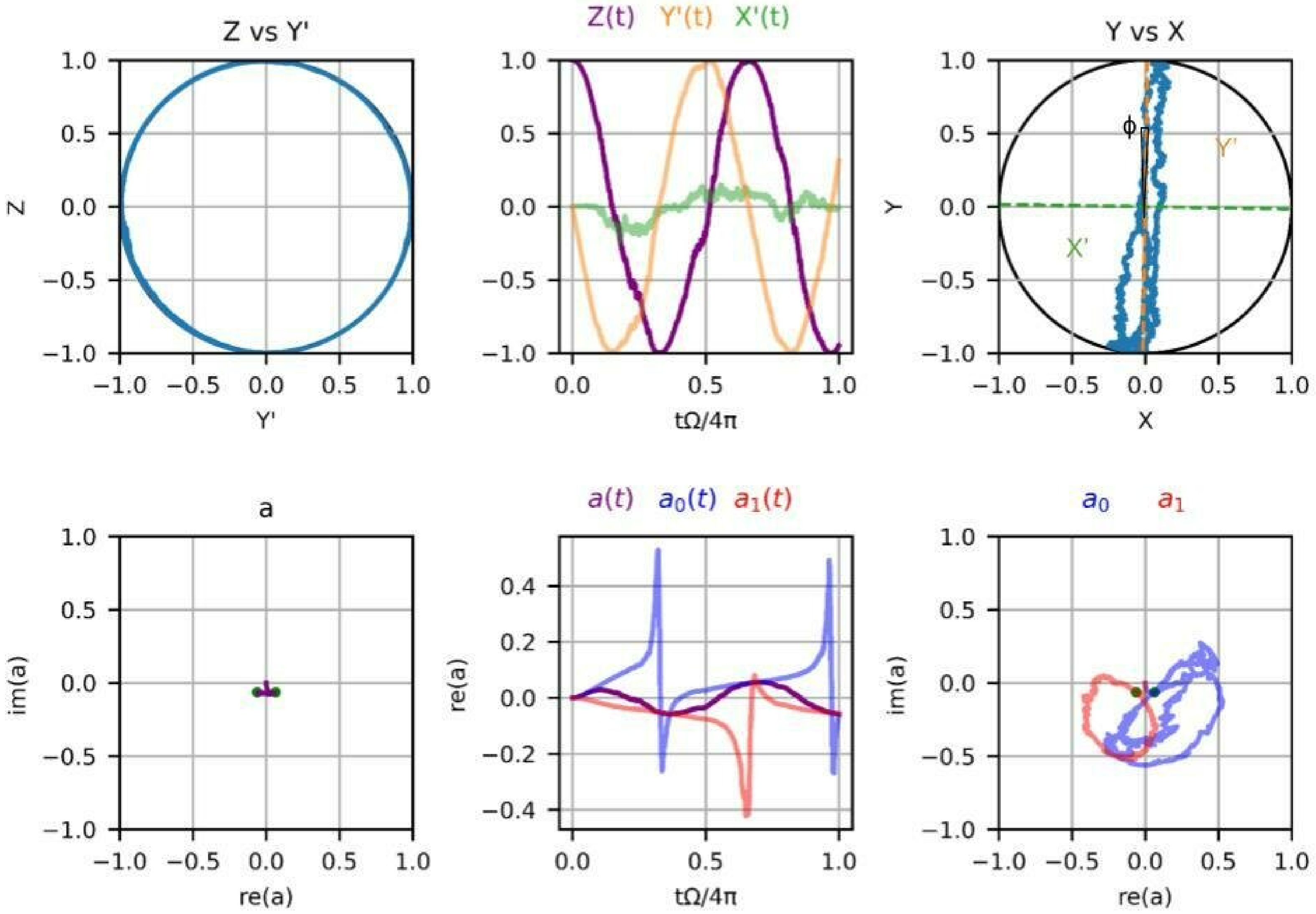
$2\Omega/\kappa = 0.8$

$2\varepsilon/\kappa = 0.125$

$\Gamma_m/\Omega = 0.02$

weak measurement

Conditioned resonator states follow weak-valued trajectories!



Plots courtesy of undergraduate Cory Panttaja: coded from scratch in python

Phase-preserving Measurements

Regime 2:

$2\chi/\kappa = 1$

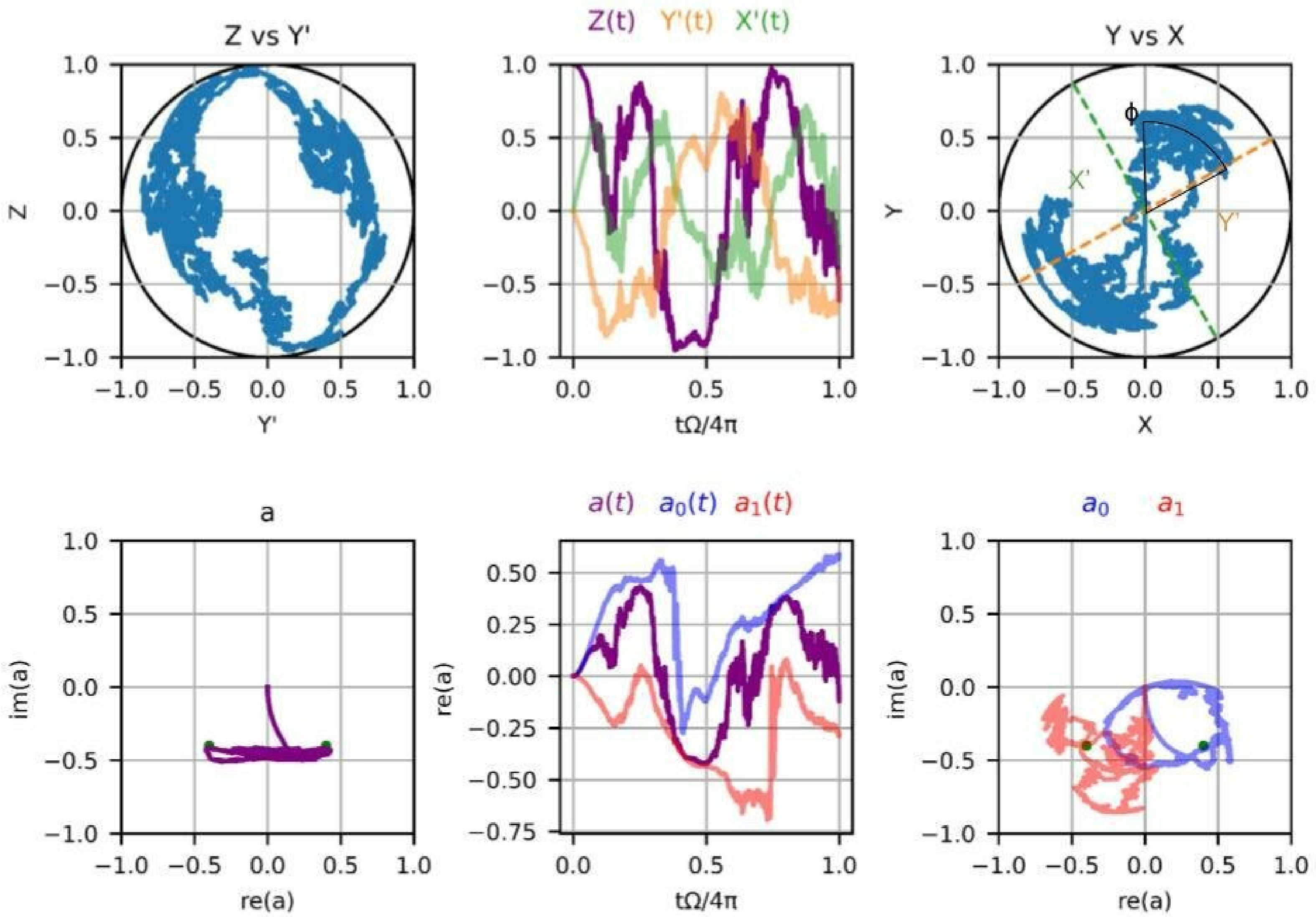
$2\Omega/\kappa = 0.8$

$2\varepsilon/\kappa = 0.8$

$\Gamma_m/\Omega = 0.8$

wishy-washy measurement

Frustrated competition between measurement and unitary dynamics



Phase-preserving Measurements

Regime 3:

$2\chi/\kappa = 1$

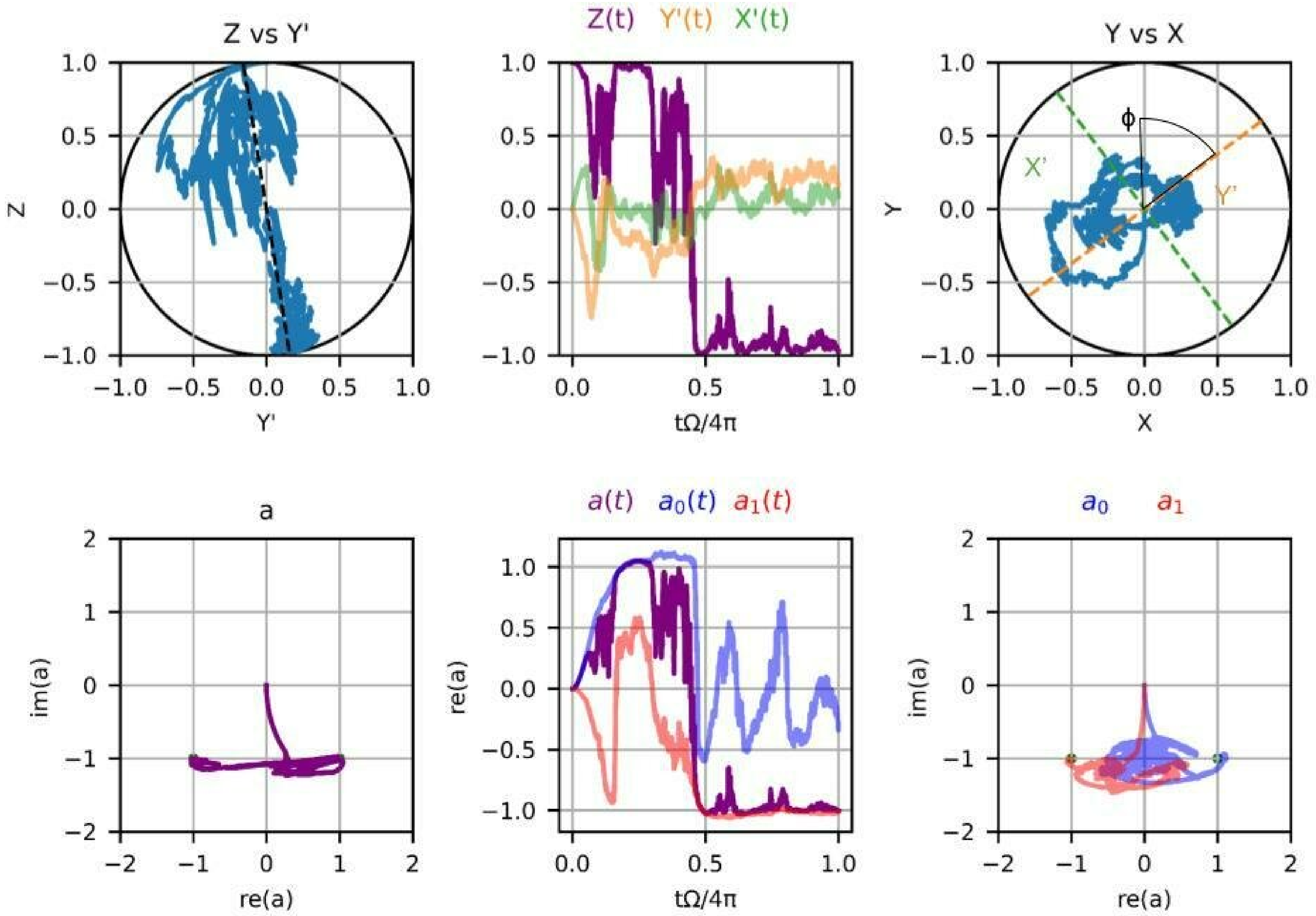
$2\Omega/\kappa = 0.8$

$2\varepsilon/\kappa = 2$

$\Gamma_m/\Omega = 5$

wimpy measurement

Tilted measurement axis and reduced measurement rate (confirms machine learning experiment)



Phase-preserving Measurements

Regime 4:

$2\chi/\kappa = 1$

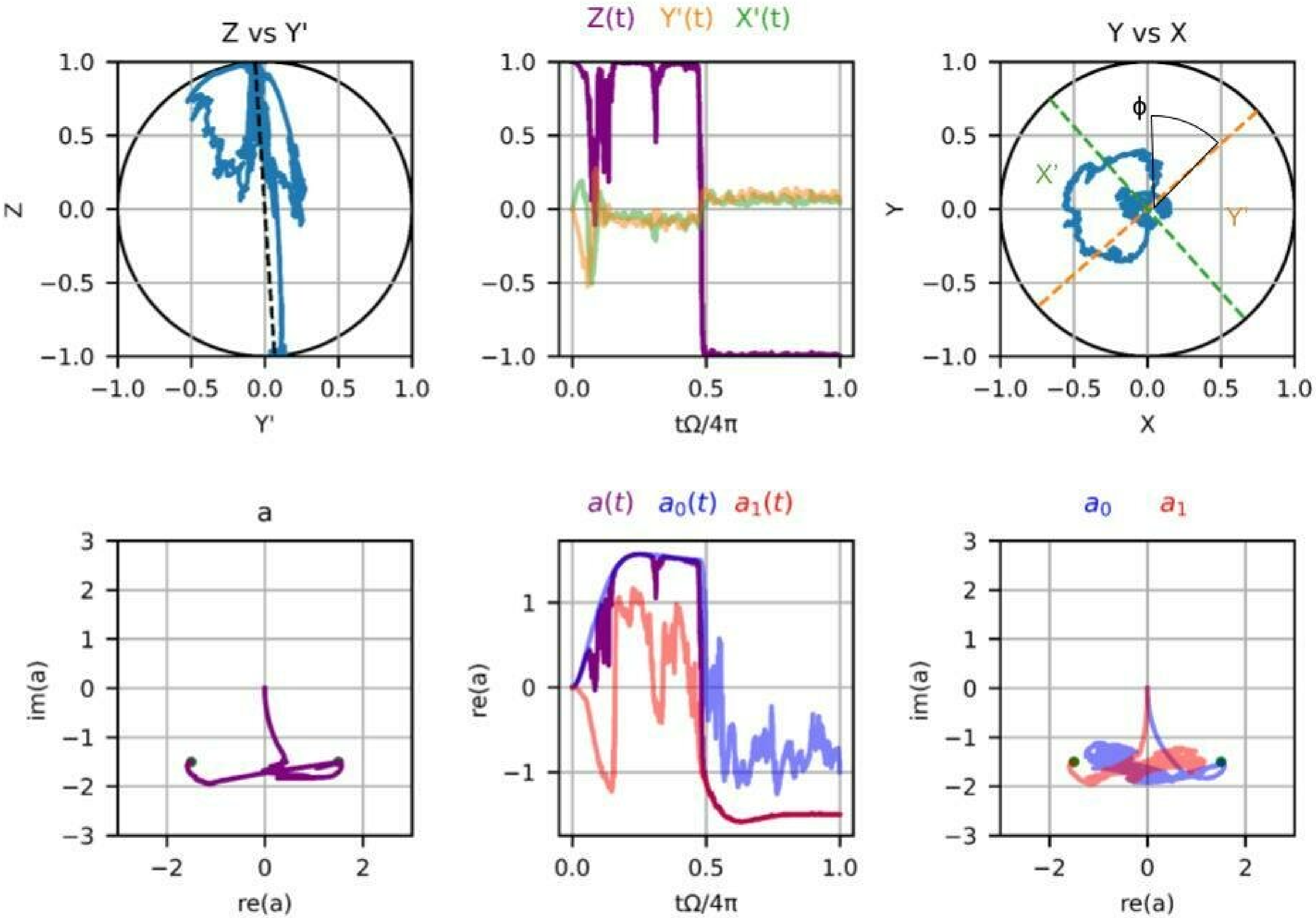
$2\Omega/\kappa = 0.8$

$2\varepsilon/\kappa = 3$

$\Gamma_m/\Omega = 11$

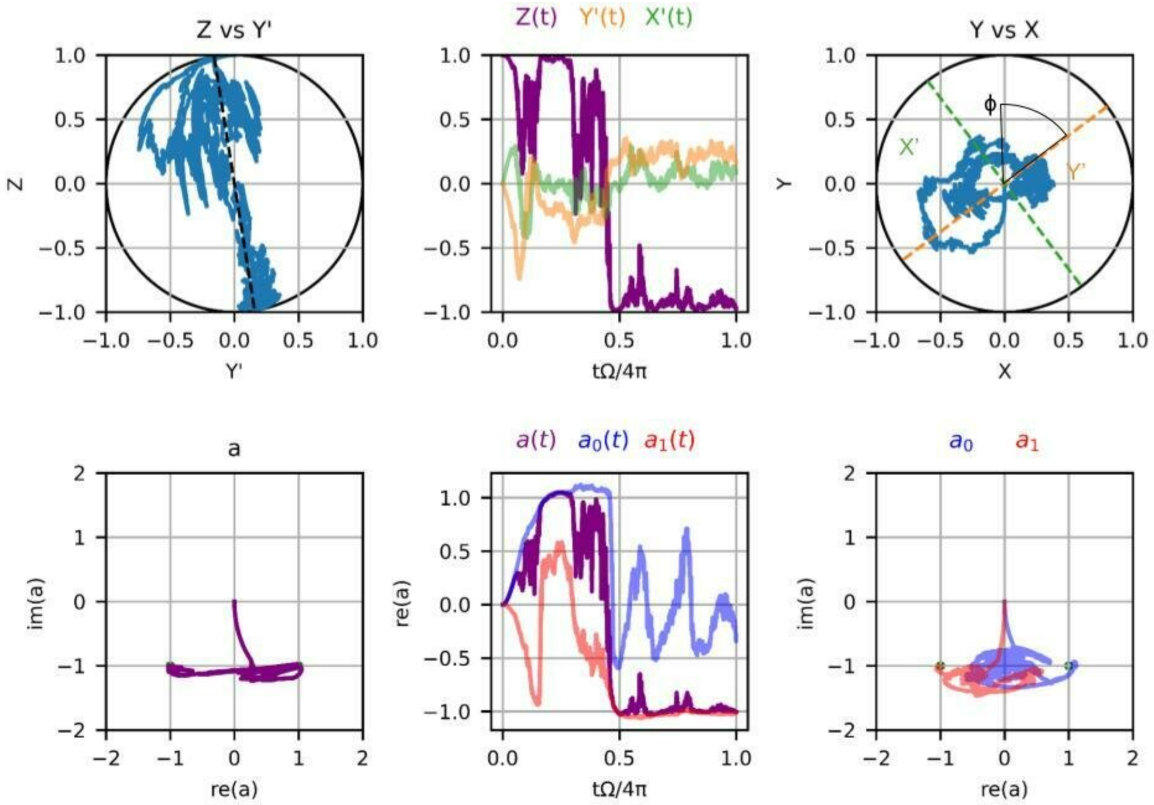
strong measurement

Zeno-pinned to measurement eigenstates, telegraphic jump behavior



Conclusions

- Superconducting qubits naturally use continuous measurements, which have much more detailed information and potential utility than binary projective measurements
- Quantum trajectories of monitored joint qubit-resonator states can now be readily simulated and analyzed using a stochastic non-Hermitian Hamiltonian
- There is very interesting behavior to explore in transient and low-probability regimes that requires this more comprehensive treatment of measurement



Thank you!

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