Simultaneous optical phase and loss estimation *revisited*: measurement and probe incompatibility

Francesco Albarelli
Università di Parma, Italy

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M.O. Bezerra, FA & R. Demkowicz-Dobrzański
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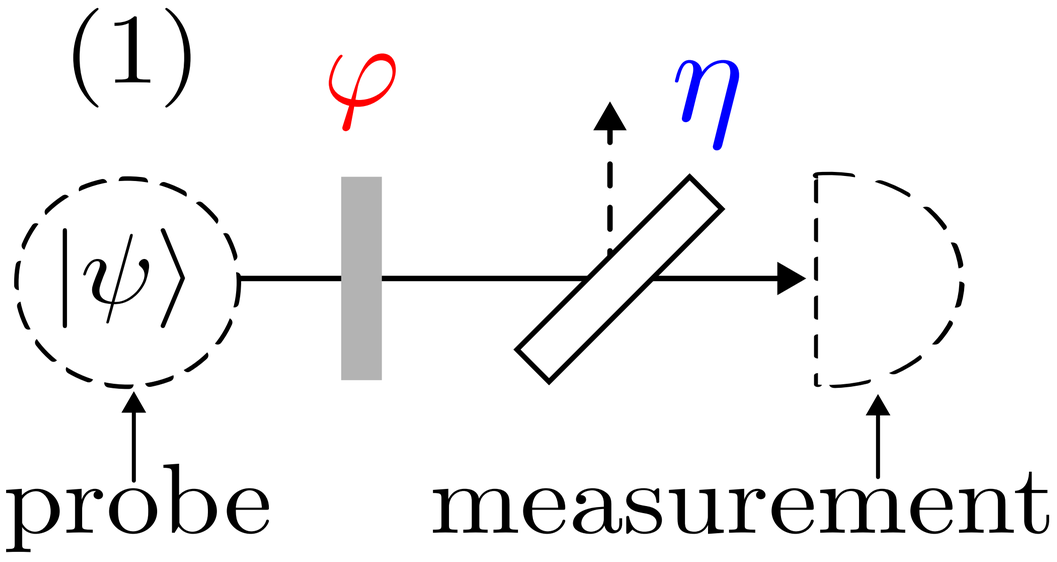
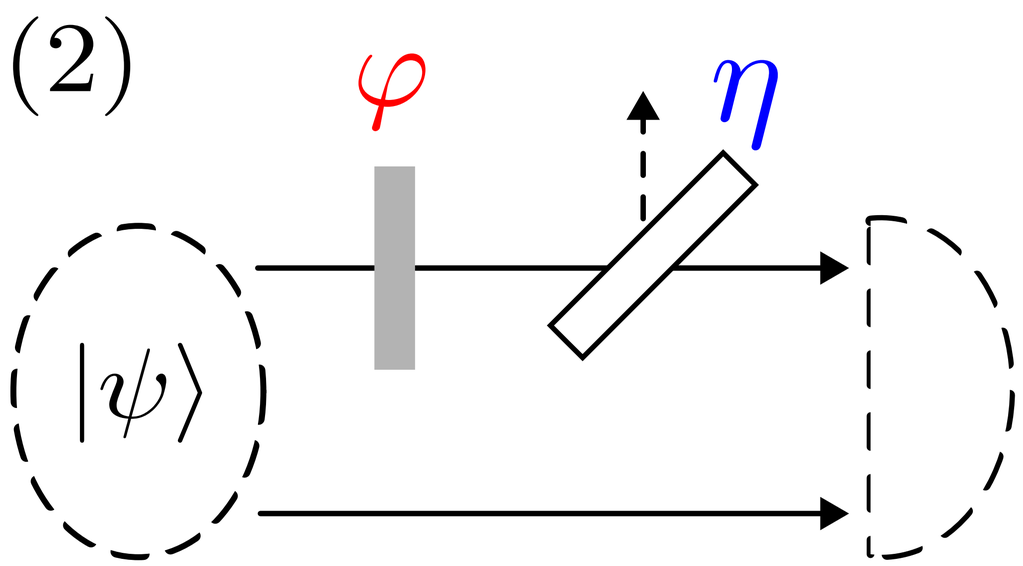
The problem

$$\hat{b} = \sqrt{\eta} e^{i\varphi} \hat{a} + \sqrt{1-\eta}\,\hat{e}$$

environment mode (vacuum)



single-mode



two-mode (w/ single-mode loss): entanglement assisted

Why “revisited”?

Paradigmatic problem with multipar. tradeoffs!

PHYSICAL REVIEW A **89**, 023845 (2014)

Tradeoff in simultaneous quantum-limited phase and loss estimation in interferometry

Philip J. D. Crowley,* Animesh Datta, Marco Barbieri, and I. A. Walmsley
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Evaluating the Holevo Cramér-Rao Bound for Multiparameter Quantum

Francesco Albarelli,[1,*] Jamie F. Friel,[1,2,†] and Animesh Datta[1,‡]
[1]*Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom*
[2]*EPSRC Centre for Doctoral Training in Diamond Science and Technology, University of Wa*
Coventry CV4 7AL, United Kingdom

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Probe Incompatibility in Multiparameter Noisy Quantum Metrology

Francesco Albarelli[1,2,*] and Rafał Demkowicz-Dobrzański[1]
[1]*Faculty of Physics, University of Warsaw, 02-093 Warszawa, Poland*
[2]*Departm*

(Received 13 M

Multiparameter quantum estimation with Gaussian states: efficiently evaluating Holevo, RLD and SLD Cramér-Rao bounds

arXiv:2504.17873

Shoukang Chang,[1,2,*] Marco G. Genoni,[2,†] and Francesco Albarelli[3,‡]
[1]*MOE Key Laboratory for Nonequilibrium Synthesis and Modulation of Condensed Matter, Shaanxi Province Key Laboratory of Quantum Information and Quantum Optoelectronic Devices, School of Physics, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China*
[2]*Department of Physics "A. Pontremoli", Università degli Studi di Milano, I-20133 Milano, Italy*
[3]*Scuola Normale Superiore, I-56126 Pisa, Italy*

Probe incompatibility

Tradeoffs in optimal probe state

(F = quantum Fisher information)

$$\max_{|\psi\rangle} \left[F_\varphi(|\psi\rangle) + F_\eta(|\psi\rangle)\right] \leq \max_{|\psi\rangle} F_\varphi(|\psi\rangle) + \max_{|\psi\rangle} F_\eta(|\psi\rangle)$$

resource N:
(average) number
of photons

$$F_\varphi^{(\max)} \xrightarrow{N\to\infty} \frac{4\eta N}{1-\eta} \qquad F_\eta^{(\max)} = \frac{N}{\eta(1-\eta)}$$

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FYI: these limits can be related through Kramers–Kronig relations

[I. Gianani, FA, A. Verna, V. Cimini, R. Demkowicz-Dobrzanski & M. Barbieri, Optica (2021)]

Probe incompatibility

Necessary conditions for joint optimality

- optimal phase estimation ~ large number uncertainty

  optimal loss estimation ~ small number uncertainty

- necessary conditions:

  i) most photons in the phase arm (loss) $\frac{\langle \hat{n}_1 \rangle}{N} \to 1$

  ii) sub-Poissonian statistics (phase) $\frac{\langle \hat{n}_1 \rangle}{\Delta^2 n} \to 0$

- possible for large N!

Measurement incompatibility

$$[\hat{L}_\varphi, \hat{L}_\eta] \neq 0$$

L = symmetric logarithmic derivative (SLD),
optimal observable for each parameter

lower bound on total error
(attainable without
meas. incompat.)

$$\frac{C^S}{C^H} \leq 1$$

attainable total error
with meas. incompat.

Results

(large photon number limit)

	N-photon states (non-Gaussian)	**Gaussian states**
(1) single-mode	Probe ✅ Measurement ❌	Probe ❌ Measurement ❌
(2) two-mode (entanglement assisted)	Probe ✅ Measurement ❌	Probe ✅ Measurement ❌

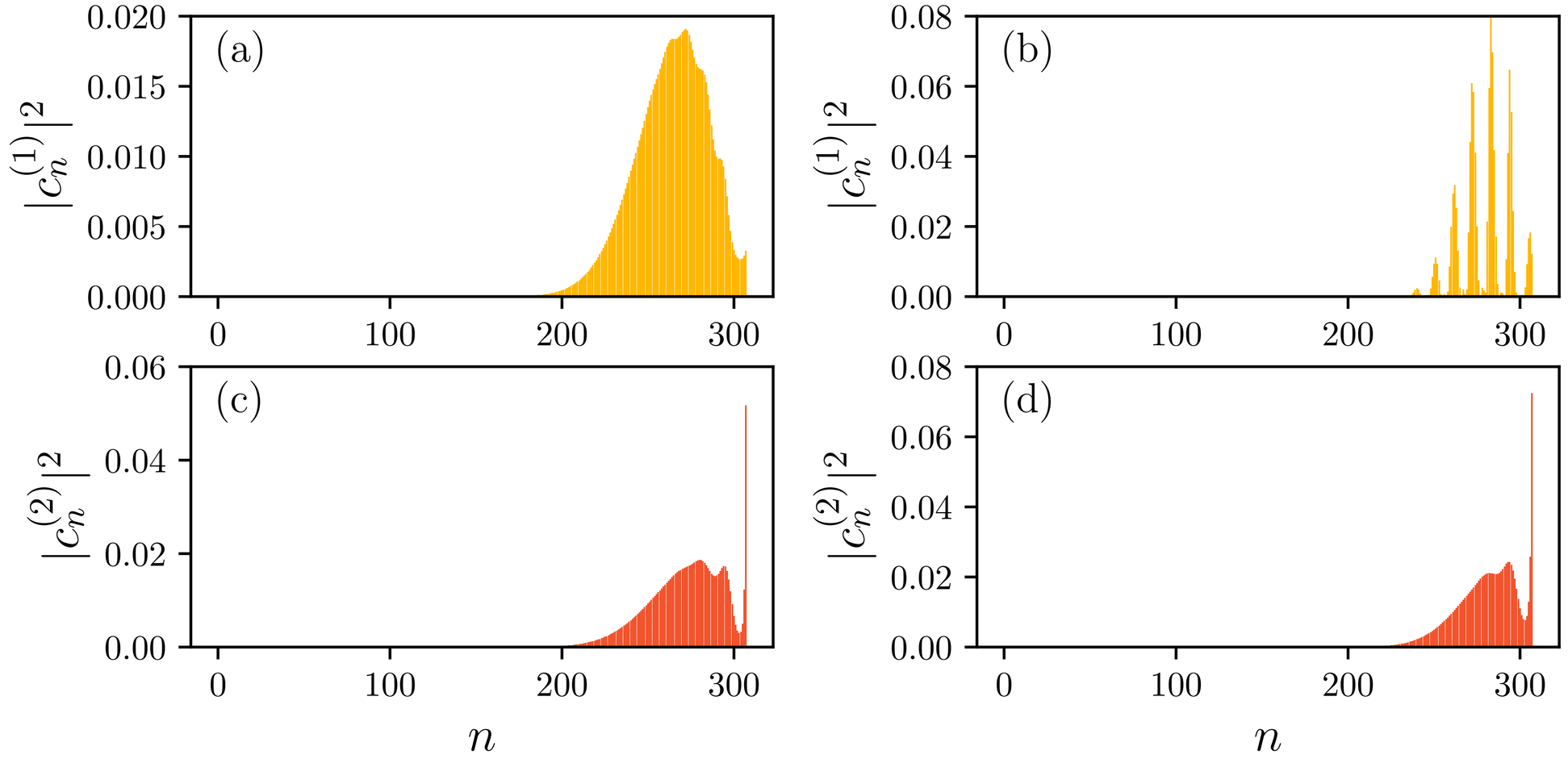
Features of optimal states

N = 307

teeth-structure: estimate
loss even with large N variance

1-mode

2-mode



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- Technical: new tool to optimize trace of QFI matrix over probe states for large N
- Non-Gaussianity for optimality in single-mode:
  somewhat similar to displacement estimation [Frigerio et al. arXiv:2504.01910]
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- Future: thermal noise (microwave regime)

Thank you for your attention!

(and feel free to get in touch if you are interested in multiparameter quantum metrology)

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