ABSTRACT FOR QUIDIQUA 2025

Discrete Wigner functions (DWFs) are used to visualise quantum states, signify nonclassicality, and support quantitative analysis in quantum information. However, in finite dimensions, many inequivalent $d \times d$ constructions coexist. This variety makes it hard to tell which features are fundamental and which are artifacts of the chosen representation. We present a unifying framework that generates *all* valid (useful) $d \times d$ DWFs from a single "parent" object on an enlarged $(2d \times 2d)$ phase space by applying a simple *stencil* through cross-correlation. We further provide conditions on these stencils (designating them as valid when the conditions are met) that ensure the creation of valid DWFs.

The framework yields several benefits. First, it provides, for fixed d, *explicit, invertible linear maps* (see figure 1) between i) any two valid DWFs representing the same operator, and ii) any two operators reconstructed from the same phase-space function but using different phase-point operator bases. As a result, these maps enable representation-independent benchmarking of quantum resource measures (e.g., negativity) and systematic comparisons of features depending on the chosen stencil, allowing one to transport properties from one representation to another. Secondly, it turns the zoo of inequivalent DWF representations into a constructible catalogue where i) every valid DWF corresponds to a given valid stencil, hence casting the DWF landscape as a stencil framework, ii) any valid DWF—for fixed d—has analogous properties (e.g., negativity, marginalisation) in every other valid DWF, and iii) it exhausts the possible definitions of a valid DWF one could ever create.

We illustrate the approach with three families. For odd dimensions, a reduction stencil recovers standard Wootters-, Leonhardt-, Gross-type DWF. For even dimensions, a coarse-grain stencil removes redundancy (a feature present in enlarged phase spaces) cleanly to produce a novel $d \times d$ DWF. Additionally, a Dirichlet-kernel stencil yields yet another DWF for odd d, yet distinct from Gross'. Together, these examples show how our framework organises the landscape and exposes representation-dependent (and independent) features of the phase-space function(s).

Implications include frame-agnostic benchmarking of resource measures, as well as clearer links between negativity, contextuality, and magic (a type of quantum resource). This also facilitates a straightforward extension to other quasidistributions, such as the Kirkwood–Dirac distribution, by relaxing the hermiticity condition. Overall, the work isolates what is truly invariant at fixed dimension and reframes "which DWF?" as an optimisable design choice for metrology, algorithms, and simulation.

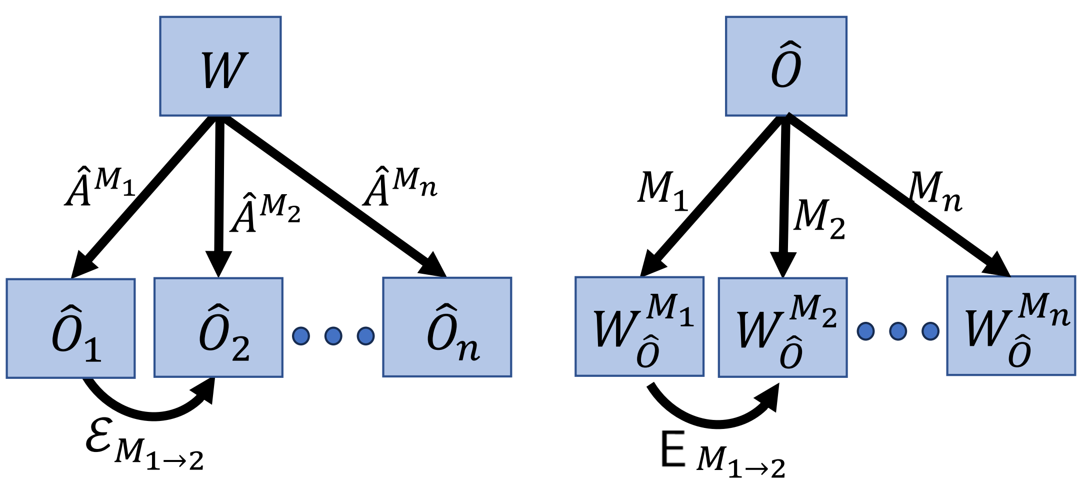

Figure 1. Left: A function on phase space, $W : \mathbb{Z}_d \times \mathbb{Z}_d \to \mathbb{C}$, can represent different operators $\hat{O}_i$ by varying the choice of stencil (M) derived phase-point operators (PPO) M-PPO $\hat{A}^{M_i}$ used to construct it. These are all related by a stencil-dependent linear map on operators, $\mathcal{E}$. Right: Analogously, each valid DWF can represent a given operator $\hat{O}$ as a distinct phase-space function, all of which are related by a stencil-dependent linear map on phase-space functions, E. These maps exist for all valid PPO frames.