Realizing Negative Quantum States with the IBM Quantum Hardware

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This study explores robust entangled states described using the framework of discrete Wigner functions. Notably, these states are known to outperform the Bell state in measures of entanglement in the presence of non-Markovian noise. Our study focuses on methods for preparing these states using quantum circuits that can be implemented on superconducting hardware and testing the efficacy of these methods on IBM's quantum device. We present quantum circuits for state preparation and validate them through tomographic reconstruction on the IBM *ibm_brisbane* device. We propose a teleportation scheme that leverages these entangled states as a resource. We believe that these entangled states have the potential to be used in place of the traditional Bell state in scenarios where non-Markovian errors are prevalent.

Entanglement is a unique ingredient in a quantum computing scheme that is pivotal to harness the full power of quantum mechanics. In addition to being a resource for speeding up computations, entanglement within quantum systems is also responsible for the robust storage of quantum information in a quantum memory [1]. Since the advent of quantum computing, considerable efforts have been directed towards attaining a profound understanding of the entanglement properties inherent in quantum systems. Classical and quantum systems can be described using a quasi-probability distribution of their configurations in the phase space. In contrast to classical systems, the quasi-probability associated with a quantum mechanical system, known as the Wigner function [2], can also assume negative values. In fact, the presence of negativity in the values of the Wigner function is widely used as a test of non-classicality in a system [3]. Although the Wigner function is conventionally applied in the context of continuous variable systems, there exists a less prevalent but potent theory concerning the discrete Wigner function (DWFs) [4, 5]. Analogously to the continuous variable scenario, the states of a system whose discrete Wigner function manifests negative values within the discrete phase space can be represented as eigenstates of the negative eigenvalues of phase-space point operators [5–7].

Noise is an imminent threat to quantum memories. The impact of noise on quantum information can be assessed by examining the open quantum dynamics of a system, which is characterized by a master equation. A general solution of the master equation that accommodates a broad range of real-world noise is articulated by the space of non-Markovian maps [8, 9]. Quantum error correction, intended to ensure the resilience of quantum information encoded within memory to noise, is frequently presumed to be effective under highly simplified assumptions about the physical noise process, such as depolarizing noise. Consequently, it remains largely uncertain yet crucial to determine whether quantum error correction and, more fundamentally, entanglement can be effectively utilized for defense against non-Markovian errors.

In a recent study, quantum states that exhibit negativity of the discrete Wigner function have been identified to be robust to a wide variety of noise in quantum systems captured by non-Markovian errors [10, 11]. Negative quantum states are defined as the normalized eigenvectors associated with the negative eigenvalues of the phase space point operator A_α at a given phase space coordinate $\alpha(q,p)$ [6, 7, 10]. The operator A_α plays a pivotal role in discrete phase space formulations, as it directly determines the discrete Wigner function (DWF) via the relation $W_\alpha = \frac{1}{d}\mathrm{Tr}(\rho A_\alpha)$, where d is the system's Hilbert space dimension. Particularly, the eigenvectors corresponding to the negative eigenvalues of A_α are known to minimize the DWF, thereby revealing signatures of quantum non-classicality [6, 10]. Using the framework developed in [4, 5], the negative quantum states are obtained for $d = 2, 3, 4$ [10, 11]. Within this framework, each line $\lambda_{i,j}$ (the j^{th} line of the i^{th} striation in discrete phase space) is assigned to a state $|\phi_{i,j}\rangle$, where these states form mutually unbiased bases (MUBs) [12]. Based on these assignments, the operator A_α is defined at each point $\alpha(q,p)$ in the discrete phase space [5], is defined as,

$$A_\alpha = \sum_{\lambda_{i,j} \ni \alpha} P_{i,j} - I, \tag{1}$$

Here, $P_{i,j}$'s are the projectors associated with the lines $\lambda_{i,j}$'s. Each of these assignments is referred to as a quantum net. In a d-dimensional system, there are $d^{(d+1)}$ such quantum nets, resulting in the $d^{(d+1)}$ distinct number of DWF

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constructions for a given density matrix ρ. These constructions must satisfy the condition that the sum of the Wigner function values W_α over a line $\lambda_{i,j}$ equals the probability of finding the system in the corresponding projector state $P_{i,j} = |\phi_{i,j}\rangle\langle\phi_{i,j}|$, i.e., $\sum_{\alpha\in\lambda_{i,j}} W_\alpha = \mathrm{Tr}(P_{i,j}\rho)$ [5]. Moreover, the eigenvector corresponding to the most negative eigenvalue of the phase space point operator A_α is referred to as the first negative quantum state and is denoted by $|NS_1\rangle$. Similarly, the second and third negative quantum states, denoted by $|NS_2\rangle$ and $|NS_3\rangle$, respectively, correspond to the normalized eigenvectors associated with the second and third most negative eigenvalues of A_α. This pattern extends to further negative eigenvalues, yielding additional negative quantum states. The two-qubit negative quantum states are recognized as optimal candidates for universal quantum teleportation using weak measurements [11] within non-Markovian noise environments. Consequently, the physical realization of these two-qubit negative quantum states is imperative, as they hold the potential to augment quantum information protocols by providing resilient resources.

Consequently, the physical realization of these two-qubit negative quantum states is imperative, as they hold the potential to augment quantum information protocols by providing resilient resources. In this work, we present methods to generate these states using operations native to superconducting hardware, specifically single-qubit gates and the CZ gate. Through quantum state tomography, we demonstrate that these states can be prepared with high fidelity on IBM's quantum hardware under realistic noise conditions. We employed various approaches to assess the noise resilience of the negative quantum states and benchmarked them against the standard Bell states. These approaches include fidelity variation analysis, phase sensitivity under $SU(2)$ rotations using quantum Fisher information (QFI), violations of Bell-CHSH inequality, and quantum information measures such as concurrence and teleportation fidelity. Our results highlight the role of negative quantum states as robust entanglement resources for a range of quantum computing and communication applications. Furthermore, we propose a teleportation circuit that utilizes one of the two-qubit negative quantum states as its entanglement resource [13].

In conclusion, we proposed methods for preparing stable entangled states, specifically focusing on two-qubit negative quantum states. We presented explicit quantum circuits to facilitate their implementation. Employing ideal and error-mitigated quantum state tomography on IBM's real quantum hardware and simulator, we validated the preparation of these states. We achieved fidelities equivalent to those of an ideal Bell state. The resilience of these states against (non)-Markovian noise is demonstrated through detailed analyses of fidelity estimation, maximal mean quantum Fisher information, optimal CHSH inequality violation, and performance in universal quantum teleportation, both with and without the application of weak measurement (WM) and quantum measurement reversal (QMR). These findings emphasize the practical relevance of negative quantum states for quantum sensing and metrology, where maintaining precision over extended timescales is critical. Furthermore, the application of WM and QMR techniques effectively extends the coherence time of these states, enhancing their viability in quantum computing, quantum key distribution, quantum teleportation, and superdense coding—areas where the preservation of quantum correlations is critical. Future directions include the development of fault-tolerant circuits for the realization of the two-qubit negative states and the generalization of these states to multiqubit architectures to design resilient quantum memories.

[1] S. J. Devitt, W. J. Munro, and K. Nemoto, Quantum error correction for beginners, Reports on Progress in Physics **76**, 076001 (2013).

[2] E. Wigner, On the quantum correction for thermodynamic equilibrium, Phys. Rev. **40**, 749 (1932).

[3] A. Kenfack and K. Życzkowski, Negativity of the wigner function as an indicator of non-classicality, Journal of Optics B: Quantum and Semiclassical Optics **6**, 396 (2004).

[4] W. K. Wootters, Picturing qubits in phase space, IBM Journal of Research and Development **48**, 99 (2004).

[5] K. S. Gibbons, M. J. Hoffman, and W. K. Wootters, Discrete phase space based on finite fields, Phys. Rev. A **70**, 062101 (2004).

[6] W. van Dam and M. Howard, Noise thresholds for higher-dimensional systems using the discrete wigner function, Phys. Rev. A **83**, 032310 (2011).

[7] A. Casaccino, E. F. Galvão, and S. Severini, Extrema of discrete wigner functions and applications, Phys. Rev. A **78**, 022310 (2008).

[8] S. Banerjee, *Open Quantum Systems: Dynamics of Nonclassical Evolution* (Springer Singapore, 2018).

[9] H. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, 2002).

[10] J. Lalita, K. Paulson, and S. Banerjee, Harnessing quantumness of states using discrete wigner functions under (non)-markovian quantum channels, Annalen der physik **535** (2023).

[11] J. Lalita and S. Banerjee, Protecting quantum correlations of negative quantum states using weak measurement under non-markovian noise, Physica Scripta **99**, 035116 (2024).

[12] W. K. Wootters and B. D. Fields, Optimal state-determination by mutually unbiased measurements, Annals of Physics **191**, 363 (1989).

[13] J. Lalita, P. Iyer, and S. Banerjee, Realizing negative quantum states with the ibm quantum hardware, arXiv preprint arXiv:2411.04608 (2024).