Efficient Variational Phase-Space Dynamics of Multi-mode Bosonic Systems in Quantum Optics

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In this presentation, we introduce a powerful new variational method [1] based on the Wigner quasi-probability distribution, capable of predicting the dynamics of multi-mode quantum nonlinear optical systems with an accuracy comparable to exact diagonalization, yet applicable to system sizes far beyond the current state of the art. The approach relies on a Variational Multi-Gaussian (VMG) ansatz for the Wigner function, where the accuracy can be systematically improved by increasing the number of Gaussian components. The variational parameters consist of the normalization coefficients, complex centers, and covariance matrices of the Gaussian functions. Their time evolution is obtained from the Dirac–Frenkel variational principle and evaluated efficiently by combining the analytic structure of Gaussian functions with automatic differentiation. A schematic overview of the method is shown in Fig. 1.

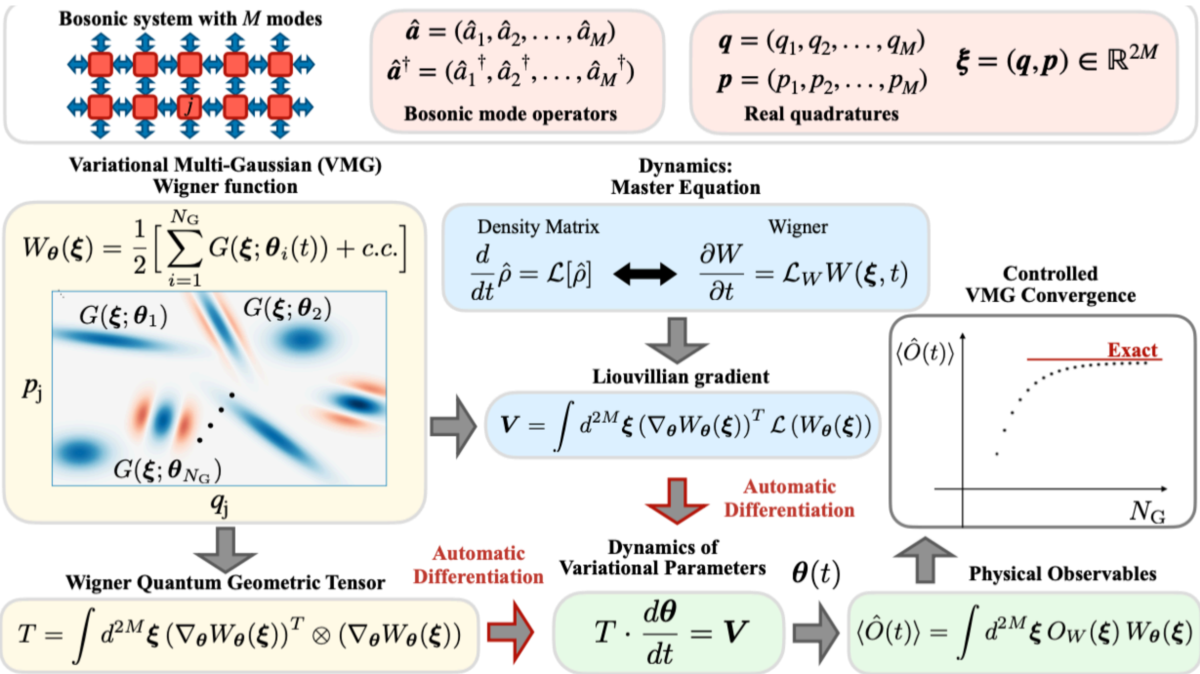

Fig1: Schematic description of our variational phase-space methods for interacting bosonic modes in the quantum regime.

This approach can accurately capture dynamics deep in the quantum regime, including states with pronounced Wigner-function negativities (see Fig. 2). Crucially, the method remains scalable to a large number of optical modes in such strongly nonclassical regimes, far beyond the current state of the art. As a first key application, we have investigated a driven–dissipative two-dimensional Bose–Hubbard lattice —physically, a network of quantum optical parametric oscillators, namely coupled nonlinear cavities subject to two-photon driving and dissipation. Our method enables the study of systems of unprecedented size (for example, 12x12 modes reported in Fig. 3), where we uncover an emergent phase transition and compute for the first time the dynamical critical exponents, which fall into the universality class of the two-dimensional quantum Ising model. This framework paves the way for exploring a wide range of complex quantum optical systems, with applications to both fundamental physics and quantum information science.

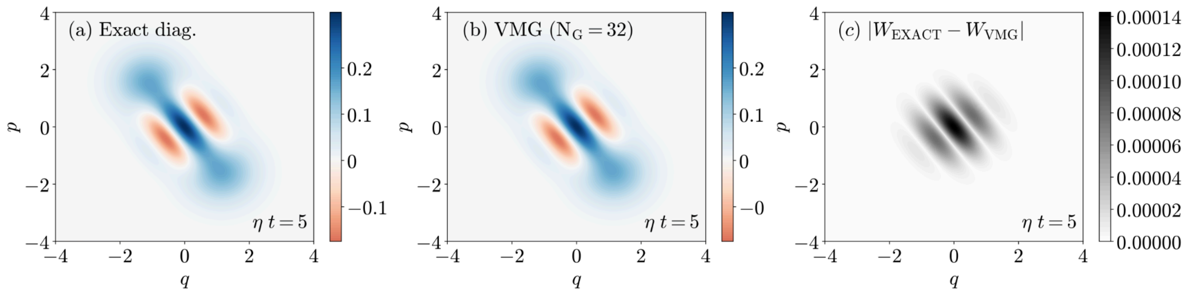

Fig2: Snapshot of the Wigner function evolution for a single Kerr resonator with a two-boson loss rate . The results demonstrate the ability to capture strongly negative Wigner functions with accuracy comparable to exact diagonalization. (a) Wigner function from exact diagonalization. (b) Variational Multi-Gaussian (VMG) Wigner function with N_G Gaussian components. (c) Absolute difference between the VMG and exact Wigner functions at a given time, starting from an initial vacuum state, showing the extreme accuracy of our variational phase-space approach.

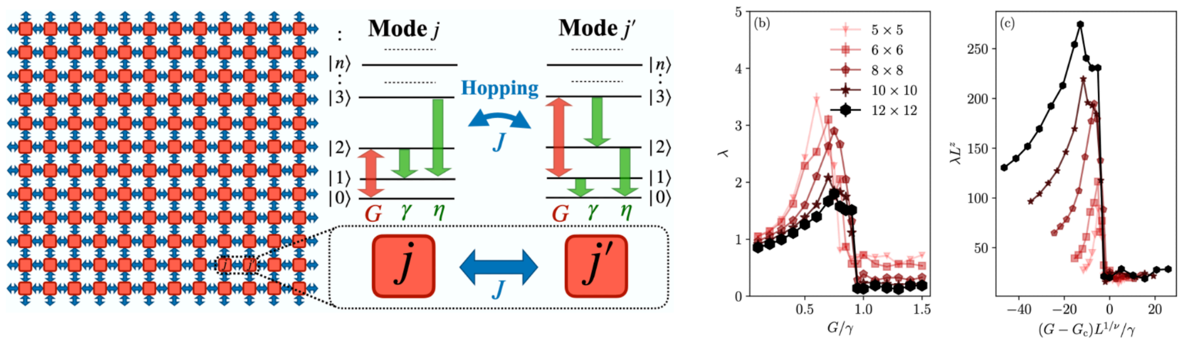

Fig3: Left panel depicts the 2D lattice of coupled quantum parametric oscillators. Each site describes a quantum optical mode with Kerr nonlinearity and subject to two-photon driving and dissipation processes. The right panel depicts the Liouvillian frequency gap, that is the asymptotic decay rate towards the steady state, versus the two-photon driving frequency G for several lattices of size L x L (up to 12x12). The finite-size scaling analysis reveals a dynamical critical behaviour consistent with the 2D quantum Ising universality class (critical exponents: β= 0.32641871, ν= 0.6299709, z = 2.0235).

BIBLIOGRAPHY

[1] J. Tosca, F. Carnazza, L. Giacomelli and C. Ciuti, *Efficient Variational Dynamics of Open Quantum Bosonic Systems via Automatic Differentiation*, arXiv:2507.14076 (2025)