Chaining Weak Measurements: a Direct Approach to Joint Measurements of Position and Momentum

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Abstract: We experimentally measure the joint value of position and momentum using two chained weak measurements, where the result of the first controls the strength of the second, enabling direct readout with a single pointer.

1. Introduction

Measuring the product of multiple observables, for example $\hat{A}\hat{B}$ for the operators $\hat{A}$ and $\hat{B}$, can be needed to characterize certain quantum effects, such as entanglement. However, it is typically not considered possible to perform such measurements on incompatible observables, such as position and momentum, since measuring one would disturb the other, resulting in a meaningless measurement. For example, measuring the position of a particle will leave it in a position eigenstate and hence subsequent measurements of momentum will be random.

The technique of weak measurement allows one to overcome this problem and perform these joint measurements. Typically this technique is considered in the formalism of von Neumann quantum measurements, where the quantum system to be measured interacts with a different known quantum state (called the pointer). After the interaction the pointer state position depends on the observable of the original quantum system being measured – the state 'points' to the value of the observable.

Key to understanding this formalism is the interaction between the measured quantum system and the pointer state. To measure an observable $\hat{A}$ of the quantum system this interaction must be the unitary shown in Eq. 1, where γ is a measure of the strength of the interaction and $\hat{P}$ is the conjugate momenta of the pointer 'position'.

$$\hat{U} = \mathrm{e}^{i\gamma\hat{A}\hat{P}} \tag{1}$$

Weak measurement then occurs when γ becomes small, allowing the quantum system being measured to be minimally disturbed. This then enables subsequent measurements on the system and allows the joint value, $\langle\hat{A}\hat{B}\rangle$, to be found by looking at the correlations between two pointers [1,2] or by using a fractional Fourier transform [3].

Here we demonstrate an alternative approach where the strength of the second pointer interaction is conditional on the result of the first measurement to create a 'chained' measurement [2]. For the observables $\hat{A}$ and $\hat{B}$, of the quantum system, this requires the interactions with the two pointers to be described by the combined unitary in Eq. 2, where the 'positions' of the two pointers are given by $\hat{q}_1$ and $\hat{q}_2$ (and the corresponding conjugate momenta are $\hat{p}_1$ and $\hat{p}_2$).

$$\hat{U} = \mathrm{e}^{i\gamma_2\hat{q}_1\hat{A}\hat{p}_2}\mathrm{e}^{i\gamma_1\hat{B}\hat{p}_1} \tag{2}$$

When there is a unitary of this form the expectation value of the second pointer 'position' is given in Eq. 3, allowing the joint value to be directly read off from the second pointer with no additional post-processing. The imaginary component of the joint value can be found by making the strength of the second interaction dependent on the conjugate momenta of the first pointer.

$$\langle\hat{q}_2\rangle = \mathrm{Re}\left[\langle\hat{A}\hat{B}\rangle\right] \tag{3}$$

One application of measuring position and momentum is that their joint value is the Dirac distribution [4] of a quantum state. This is informationally equivalent to the density matrix to represent a mixed state, and hence enables truly direct measurements of a mixed state to be performed.

2. Experiment

We measure the joint value of the transverse position x and momentum p_x of a photon. In our experiment (Fig. 1) the first pointer is the orthogonal transverse position y which is coupled to a particular x position using a glass sliver tilted into the beam, as shown in Fig. 2a. To perform measurements at different values of x the sliver is translated in the x direction.

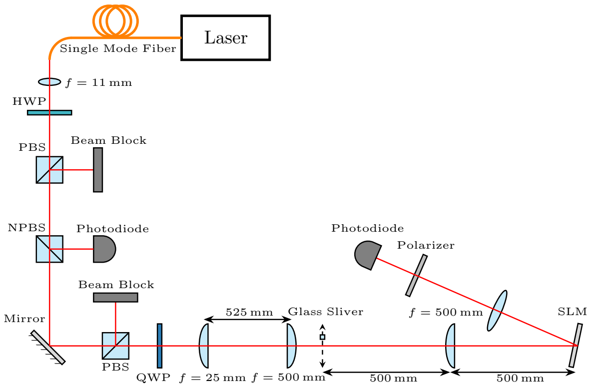
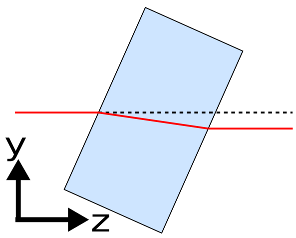
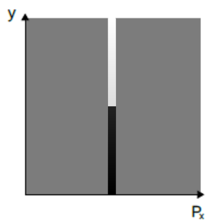


Fig. 1: The experimental setup to perform a joint measurement of position x and transverse momentum p_x.

The second pointer is the photon polarization which is coupled to the p_x momentum with a strength dependent on the value of the first pointer (y) using a spatial light modulator (SLM). The SLM is placed in the Fourier plane of the glass sliver using a lens, so each horizontal position on the SLM corresponds to a p_x at the glass sliver. The second pointer is initialized with circular polarization and the action of the SLM is to rotate the polarization about the H/V axis of the Bloch sphere by a programmable amount at each pixel of the SLM. By displaying a binary 'split' pattern on the SLM this causes the amount of rotation (strength of the interaction) to depend on the value of the first pointer y. By displaying this binary pattern in a 'sliver' (shown in Fig. 2b), one value of p_x will couple to the 2nd pointer with one of two possible strengths, conditional on the result of the first pointer. Observing the amount the polarization has rotated after the SLM will then give a direct measurement of the joint value of x and p_x.



(a) A tilted glass sliver displaces the beam in the y direction.



(b) A binary split pattern on the SLM creates an interaction whose strength is one of two possible values, depending on the value of the first pointer.

Fig. 2: The two interactions are carried out by a tilted glass sliver and a binary split sliver on a SLM.

3. Conclusion

To summarize, we demonstrated that chaining weak measurements allows the joint value of two incompatible observables (position and momentum) to be found, and that this value can be directly read from a single pointer.

References

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