Positivity of the Kirkwood-Dirac distribution associated with Fourier transform for continuous variable systems

Matéo Spriet[1]

[1]Univ. Lille, CNRS, Inria, UMR 8524, Laboratoire Paul Painlevé, F-59000 Lille, France

The Kirkwood-Dirac (KD) distribution was introduced in the early stages of quantum theory and has gained recently renewed interest. It is mainly studied in the literature for discrete variable quantum systems. In this talk, we present recent results concerning the Kirkwood-Dirac distribution associated with the Fourier transform for continuous variable systems. It is a generalization of the KD distribution associated with position and momentum operators. The associated classical fragment of quantum mechanics is the set of states having a positive KD distribution, together with the set of observables having a real KD distribution. We explicitly describe this classical fragment in two cases: the particle living on the real line and the particle living on a circle.

In the case of the line, the KD distribution of any Hilbert-Schmidt self-adjoint operator admits a non-zero imaginary part, implying that the associated classical fragment is trivial if we consider only normalizable states and Hilbert-Schmidt observables. However, there exists KD-positive non-normalizable states, and we provide the exhaustive list of the non-normalizable KD-positive pure states.

In the case of the circle, we show that the classical fragment consists of states and observables that are diagonal in the Fourier basis. This implies that any KD-positive state can be expressed as a convex combination of KD-positive pure states.

The above results are special instances of a recent work on the Kirkwood-Dirac distribution associated with the Fourier transform on (second countable) locally compact abelian groups [2]. This work is a continuation of the work started for the Kirkwood-Dirac distribution defined for the Fourier transform on finite abelian groups [1].

References

[1] Stephan De Bièvre, Christopher Langrenez, and Danylo Radchenko. *The Kirkwood-Dirac Representation Associated to the Fourier Transform for Finite Abelian Groups: Positivity.* Jan. 21, 2025. Pre-published.

[2] Matéo Spriet. *Characterizing the Kirkwood-Dirac Positivity on Second Countable LCA Groups.* July 31, 2025. Pre-published.