Characterization of Bound Magic states through the Kirkwood-Dirac distribution

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(Dated: September 4, 2025)

Classical algorithms can simulate efficiently quantum circuits throughout which the quantum state has a positive quasiprobability representation. Recently, it was shown that the $\mathbb{Z}_2^2$ Kirkwood–Dirac (KD) distribution can simulate efficiently the operations allowed by CSS error-correcting codes. In this talk, we map out the geometry of positive states of the $\mathbb{Z}_2^2$ KD distribution. We present both analytical and numerical results. In particular, we uncover a volume of previously unknown bound magic states, states that are not stabilizer mixtures, yet have positive KD distribution. Our analysis reveals that these bound states occupy approximately 0.69% of the two-qubit state space. Compared to previous results, this constitutes a 15% increase in the volume of input states that enable efficient classical simulation through CSS circuits.