Abstract: Majorization theory for quasiprobabilities

Twesh Upadhyaya [1 2] Zacharie Van Herstraeten [3] Jack Davis [3]
Oliver Hahn [4] Nikolaos Koukoulekidis [5 6 7] Ulysse Chabaud [3]

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Majorization theory is a powerful mathematical tool to compare the disorder in distributions, with wide-ranging applications in many fields including mathematics, physics, information theory, and economics. While majorization theory typically focuses on probability distributions, quasiprobability distributions provide a pivotal framework for advancing our understanding of quantum mechanics, quantum information, and signal processing. Here, we introduce a notion of majorization for continuous quasiprobability distributions over infinite measure spaces. Generalizing a seminal theorem by Hardy, Littlewood, and Pólya, we prove the equivalence of four definitions for both majorization and relative majorization in this setting. We give several applications of our results in the context of quantum resource theories, obtaining new families of resource monotones and no-goes for quantum state conversions. A prominent example we explore is the Wigner function in quantum optics. More generally, our results provide an extensive majorization framework for assessing the disorder of integrable functions over infinite measure spaces. https://arxiv.org/abs/2507.22986

[1]Joint Center for Quantum Information and Computer Science, University of Maryland and NIST, College Park, MD 20742, USA
[2]Department of Physics, University of Maryland, College Park, MD 20742, USA
[3]QAT team, DIENS, École Normale Supérieure, PSL University, CNRS, INRIA, 45 rue d'Ulm, Paris 75005, France
[4]Department of Basic Science, The University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo 153-8902, Japan
[5]Duke Quantum Center, Duke University, Durham, NC 27708, USA
[6]Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA
[7]Department of Physics, Duke University, Durham, NC 27708, USA