Witnessing genuine continuous-variable contextuality and Bell non-locality

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Contextuality and Bell non-locality represent fundamental non-classical features of quantum systems that serve as resources for quantum computational and communicational advantages. Bell non-locality is a special case of contextuality. In particular, for multipartite scenarios, with only local measurement allowed, the contextual fraction – a quantifier of contextuality – is equivalent to a normalised Bell inequality violation.

Although the original EPR paradox was formulated in terms of continuous variables, most studies of these phenomena have focused on discrete-variable scenarios. Recently, a continuous-variable formalism of contextuality was introduced in [1] along with a method to quantify the amount of contextuality in a physical experiment. This method is a continous-variable generalisation of the contextual fraction as a solution of an infinite-dimensional linear program. A hierarchy of semi-definite programs converging to the contextual fraction is also provided in [1]. Yet, no explicit experiment that would witness a genuine continuous-variable violation of a Bell inequality was provided in [1].

Our work uses this method and provide explicit example to probe the genuine continuous-variable Bell inequality violation. With respect to previous strategies employed on continuous variable settings, here we do not need to hand pick a binary discretization of the data, but rather rely directly on the histograms obtained from homodyne detection, with a homogeneous binning. Using this strategy, we find out that, for some previously studied empirical models [2-4], the Bell inequality violations obtained, when normalized to unity, are exactly equivalent to the contextual fraction, meaning they are optimal for the homodyne settings and states that they consider. Notably, these examples include two-photon subtracted states, which are within the reach of state of the art quantum optics experiments.

References

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