

Symmetry enhanced classical simulation of Clifford dominated circuits

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Classical simulation of quantum circuits plays a crucial role in validating quantum hardware and delineating the boundaries of quantum advantage. Among the most effective simulation techniques are those based on the stabilizer extent, which quantifies the overhead of representing non-Clifford operations as linear combinations of Clifford unitaries. It quantifies how "resourceful" an operation is for quantum computation. It is connected to optimal 1-norm expansions of operations over the Clifford group, and it is linked to quasiprobabilities in a pure-state picture. Specifically, the L1-sampling of an optimal stabilizer extent decomposition for a gate can be used to find sparse Clifford representations for classical simulation.

However, finding optimal decompositions rapidly becomes intractable as it constitutes a superexponentially large optimization problem. In this work, we exploit symmetries in the computation of the stabilizer extent, proving that for real, diagonal, and real-diagonal unitaries, the optimization can be restricted to the corresponding subgroups of the Clifford group without loss of optimality. This "strong symmetry reduction" drastically reduces computational cost, enabling optimal decompositions of unitaries on up to seven qubits using a standard laptop--far beyond previous two-qubit limits. Additionally, we employ a "weak symmetry reduction" method that leverages additional invariances to shrink the search space further. Applying these results, we demonstrate exponential runtime improvements in classical simulations of quantum Fourier transform circuits and measurement-based quantum computations on the Union Jack lattice, as well as new insights into the non-stabilizer properties of multi-controlled-phase gates and unitaries generating hypergraph states. Our findings establish symmetry exploitation as a powerful route to scale classical simulation techniques and deepen the resource-theoretic understanding of quantum advantage.