Information for negative probabilities and entropic uncertainty in phase space

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Lying at the heart of information theory, entropy functionals quantify the statistical uncertainties and correlations of random variables. However, they involve logarithms of the underlying probability distributions, thus rendering information theory heavily reliant on non-negative probabilities. This poses significant challenges when analyzing information in quantum systems using the Wigner W-distribution, which is well known to yield negative values for nonclassical states.

Here, we propose extensions of standard entropic measures to accommodate negative probabilities by accounting for their sign with extra bits. We demonstrate that our definitions preserve, in a generalized sense, many fundamental properties of standard entropies, which allows us to prove a generalized variant of the Wigner entropy conjecture. Further, we put forward tight entropic uncertainty relations (EURs) for mixed states and conjecture a novel EUR that is tight for all Gaussian states, whether pure or mixed.