Information for negative probabilities and proof of the Wigner entropy conjecture

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Lying at the heart of information theory, entropy functionals quantify the statistical uncertainties and correlations of random variables. However, they involve logarithms of the underlying probability distributions, thus rendering information theory heavily reliant on non-negative probabilities. This poses significant challenges when analyzing information in quantum systems via the Wigner W-distribution, which is well-known to produce negativities for non-classical states.

Here, we propose extensions of standard entropic measures to accommodate negative probabilities by accounting for their sign with extra bits. We demonstrate that our definitions preserve fundamental properties of standard entropies in a generalized sense, including subadditivity and the entropy power inequality. This allows us to derive entropic uncertainty relations for the family of s-ordered phase-space distributions, which, for s=0, yields a proof the (generalized) Wigner entropy conjecture for arbitrary quantum states.