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JOINT CENTER FOR QUANTUM INFORMATION AND COMPUTER SCIENCE

Majorization theory for quasiprobabilities

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NSERC CRSNG

QuiDiQua^3

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What is majorization?

- Fundamental way to quantify the disorder in a distribution
- Many applications across mathematics, physics, economics
- Simple example, which biased coin is more predictable?

$$\begin{pmatrix} 0.8 \\ 0.2 \end{pmatrix} \quad \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$$

Extending majorization's reach

- Well-studied for probability distributions
- What about quasiprobabilities?
  - Real-valued, normalized and possibly negative distributions
- Finite measure spaces ✅
- Infinite measure spaces **?**

		Measure Space	
		Finite	Infinite
Distribution	Probability	✅	✅
	Quasiprobability	✅	**?**

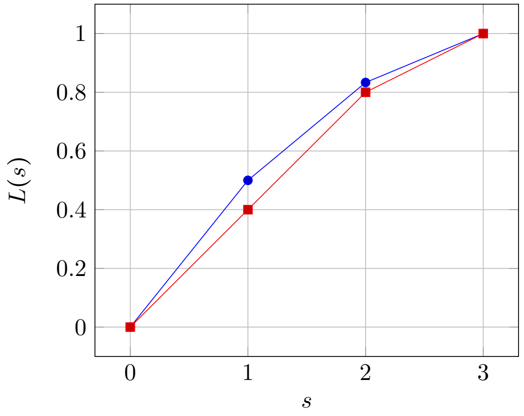
Our results

- We extend the definition of majorization to real-valued, normalized distributions over infinite measure spaces
- We prove the equivalence of the several different characterizations is retained
- Our definition recovers the usual ones in simpler cases
- Applying our general framework to CV Wigner distributions, we obtain new state conversion bounds and new families of monotones for quantum resource theories with various sets of free operations

*And relative majorization

Equivalent Characterizations of Majorization

$$\begin{pmatrix} 1/2 \\ 1/3 \\ 1/6 \end{pmatrix} \succ \begin{pmatrix} 2/5 \\ 2/5 \\ 1/5 \end{pmatrix}$$



- Lorenz Curves - Predictability

$$\begin{pmatrix} 2/5 \\ 2/5 \\ 1/5 \end{pmatrix} = \begin{pmatrix} 2/5 & 3/5 & 0 \\ 1/2 & 2/5 & 1/10 \\ 1/10 & 0 & 9/10 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/3 \\ 1/6 \end{pmatrix}$$

- Doubly Stochastic Operators - Mixing

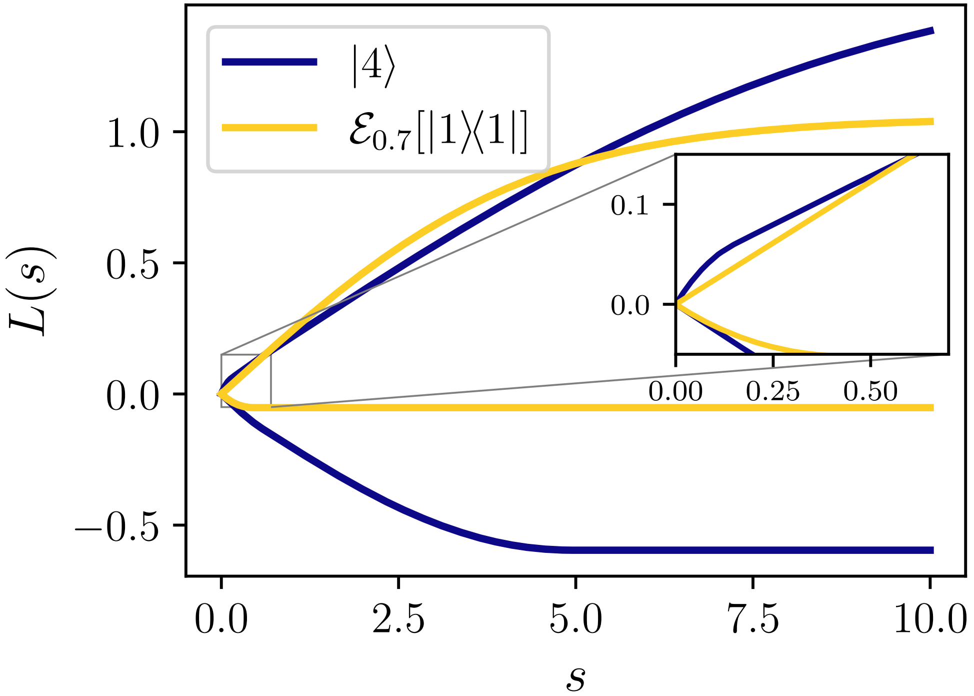
$$\Phi(2/5) + \Phi(2/5) + \Phi(1/5) \geq \Phi(1/2) + \Phi(1/3) + \Phi(1/6)$$

- Schur-Concave Functionals - Entropy

Overcoming infinite zero plateau

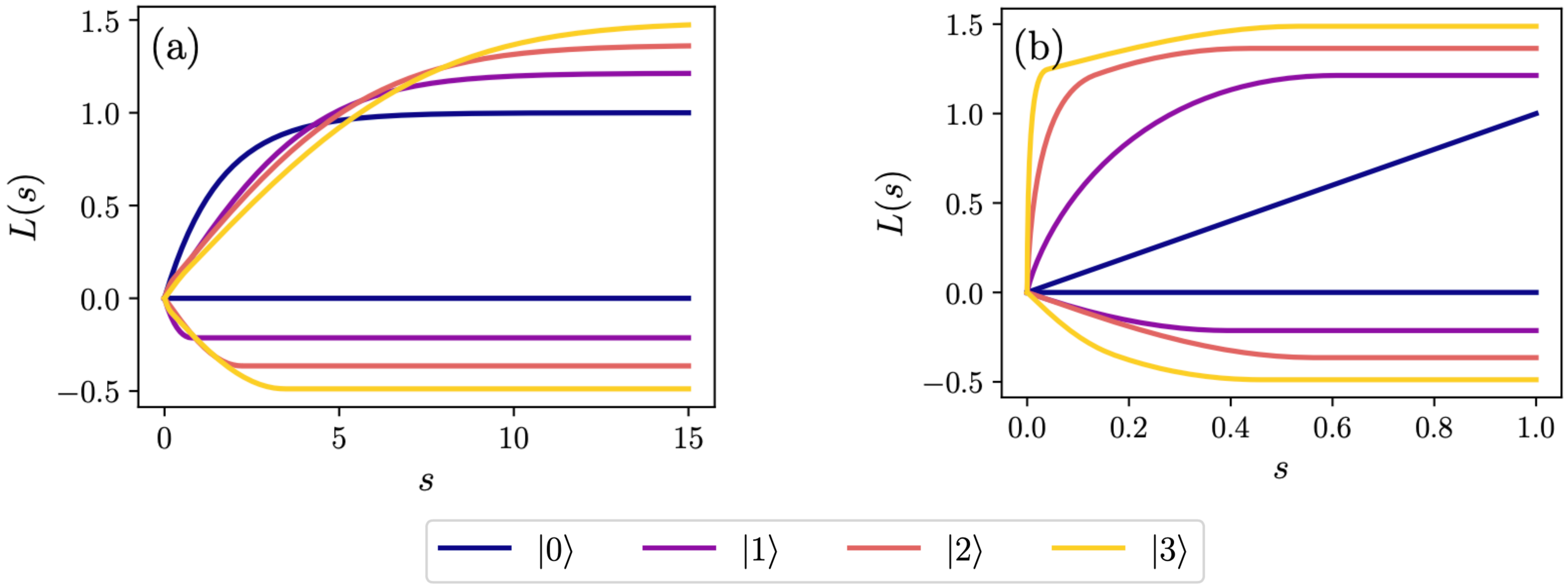
- Informally, an integrable function over an infinite measure space must be close to zero most of the time
- Arranging values from largest to smallest pushes all negative values ‘to infinity’, i.e. loses all information about them
- Resolution is to use *both* increasing and decreasing rearrangements – i.e. both Lorenz curves

Wigner Lorenz Curves



- Rules out conversion with amplifying Gaussian channels, mixtures of such channels, dilations, etc.
- Not flagged by other resource monotones

Wigner Lorenz Curves *(cont.)*



Summary and future directions

- Introduce a framework for majorization of quasiprobabilities over infinite measure spaces
- Apply it to Wigner functions to derive new restrictions on state conversions under various classes of operations
- Questions to this audience
  - Study Lorenz curves of more interesting quantum states
  - Apply to other quasiprobability representations – what are the positive channels in that representation?
  - Does this framework help assess the usefulness of states in other contexts, for example quantum speedup ?
- Thanks and happy to take questions!

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